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THE EFFECT OF RED AND INFRA-RED ON THE DECAY OF PHOSPHORESCENCE IN ZINC SULPHIDE

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The effect of long-wave radiation upon phosphorescence has been studied by nearly all investigators of phosphorescent phenomena. The action of red or infra-red is to hasten the decay of phosphorescence, a phenomenon used by Draper and others to map infra-red spectra. The manner of action differs with the phosphorescent substance. With many the incidence of long-wave energy causes a flashing-up of luminescence, after which follows a rapid decay. With some others this preliminary flash is apparently absent. Among these latter is zinc sulphide, variously known as sidot-blende or "Emanationspulver," depending on its source or method of preparation. Among recent observers who have studied this substance may be mentioned Dahms,[1] Nichols and Merritt,[2] Werner,[3] and Lenard.[4] Various phases of the behavior of this sulphide have occupied the attention of each of these investigators but they have been in general agreement until recently that the flashing-up of phosphorescence is absent. But Lenard, who distinguishes several varieties of phosphorescent zinc sulphide,

[1] *Annalen der Physik*, **13**, 425, 1904.

[2] *Physical Review*, **25**, 362; **23**, 37; **22**, 279; etc.

[3] *Annalen der Physik*, **24**, 164, 1907; **30**, 257, 1969.

[4] *Ibid.*, **31**, 641, 1910.

mentions in his latest publication that even in the variety most susceptible to the extinguishing action of red light a very brief flash may be seen if properly looked for.

This phenomenon is of some importance in developing a satisfactory theory of phosphorescence. It was originally held, by Becquerel, that the total quantity of radiation given out by a phosphorescent mass during decay is a constant. The effect of red radiation is simply to accelerate the giving up of the stored energy; consequently the flashing-up is a necessary process to maintain the constancy of the total radiation. The existence of substances not exhibiting this effect demands the discarding or the

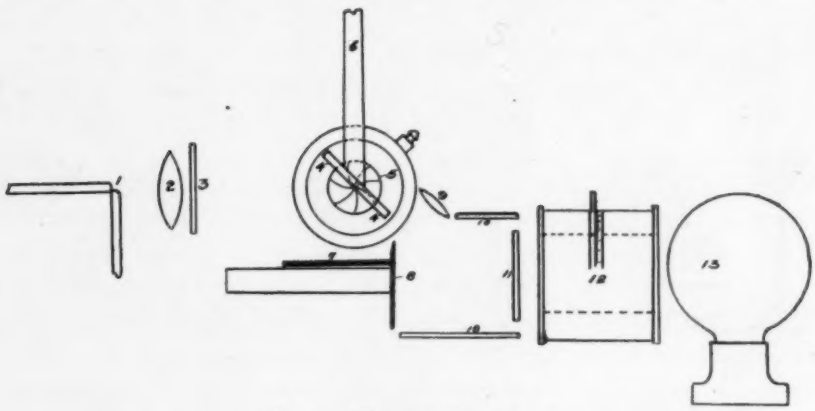


FIG. 1.—Diagram of apparatus

modifying of this idea. A knowledge of the true character of the behavior of these substances under the influence of long-wave energy is necessary to indicate the requisite modification of this original theory or the substitute for it.

The experiments described in this paper show that in zinc sulphide the flashing-up phenomenon is clearly present, with the peculiarity that it becomes striking only upon the lapse of some time after the beginning of the decay of phosphorescence. The investigation started by an observation made during a test of zinc sulphide for its use in the detection of infra-red radiation in the fire-fly. At first it appeared that a flash of red light extinguished the phosphorescence only if it occurred within a few seconds after

the beginning of decay. Later during the process of decay a similar brief exposure to red light caused no extinction. But further observation showed that the extinguishing effect was apparently merely delayed in starting. If given time, the extinction produced by an exposure five minutes after excitation would attain as great a value as that from a similar exposure immediately after excitation.

For the exact study of this phenomenon a new arrangement of photometric apparatus was employed, possessing some decided advantages over any heretofore published. The apparatus is shown diagrammatically in Fig. 1: 7 is the surface of zinc sulphide lying horizontally; 8, a thin metal sheet whitened with magnesium oxide and standing vertically. Excitation of the sulphide is produced by the light of a carbon arc through blue glasses (transmitting no red) at 3, the light being reflected by the platinum-faced mirror, 4. This mirror can be turned about a vertical axis, 6, so as to receive when desired the red or infra-red radiation from a 100 c.p. carbon lamp behind 5. At 5 is a "Volute" photographic shutter for regulating the time of exposure to red or infra-red. The vertical surface at 8 is illuminated by the incandescent lamp at 13 whose light passes through the Brodhun sector, 12, and through the colored glasses, 11. By changing the voltage of the lamp and by altering the opening of the sector, a very large range of illuminations is obtainable. The observer looking through the lens, 9, sees as photometric field the zinc sulphide surface, 7, and the vertical sheet, 8. Holding his hand upon the sector he is able to make photometric settings rapidly, while an assistant reads the times and the opening of the sector. By this means such phenomena as the flash-up of luminosity may be followed and measured accurately. By the addition of a chronograph and an automatic recorder this apparatus should make it possible to secure measurements just as fast as the eye and hand can make settings. For the present work sufficient speed was attained without these additions.

The first measurements made with this apparatus revealed a more interesting phenomenon than had been suspected. The initial experiments were to determine the effect of flashes of one second's duration of the light from the incandescent lamp through

a ruby glass (carefully tested to insure no blue or ultra-violet transmission). The phosphorescent surface was first reduced to a standard condition according to the procedure of Nichols and Merritt by five minutes' exposure to red, followed by five minutes' rest. The duration of excitation was uniformly two minutes (except where noted otherwise) with a constant current through the arc. Under these conditions, the initial brightness of phosphorescence and the rate of decay were very satisfactorily uniform. The results are shown in Fig. 2. It appears that after a certain lapse of time, zinc sulphide does show the flashing-up phenomenon. The apparent lag in the extinguishing effect which first attracted attention appears to be merely the intermediate stage between the early condition where rapid extinction occurs and the later condition where increased brightness precedes extinction.

A long series of observations has been made on this phenomenon as it occurs under varied conditions of excitation and exposure to long waves. In some of these the exposure to long-wave energy consisted of flashes of definite length of red light, in others to flashes of infra-red (carbon disulphide and iodine cell

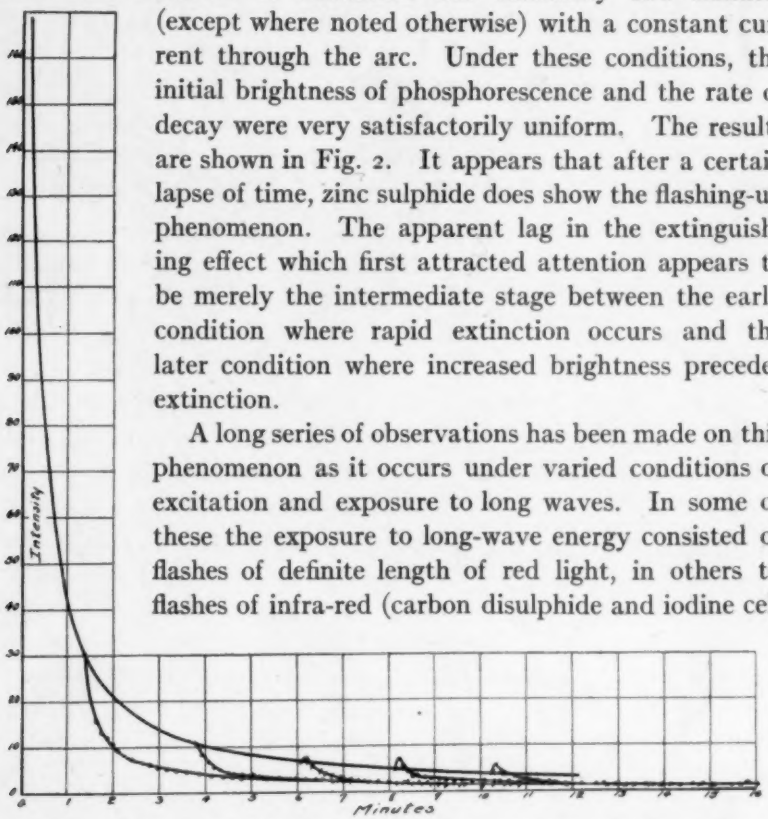


FIG. 2.—Effect of red light on the phosphorescence of zinc sulphide at different periods during decay. Full excitation 2 minutes; one second flashes of red.

added to ruby glass), in still others the exposure to infra-red once started was allowed to continue. This last procedure of course is only compatible with the observation and measurement of phosphorescence if the extinguishing radiation is invisible. The particular mode of working will be noted as the various experiments are described. Those first performed were with red light while

the later ones demanded infra-red, and in making the change no qualitative difference in the effect of the two was found.

Effect of varied quantity of red radiation.—The amount of flashing-up and the shortest interval after the beginning of decay that this flash-up can be noticed were studied by interposing different neutral-tint absorbing screens over the red light. Flashes were made on the sulphide surface at approximately one, three, five, eight, and fifteen minutes after excitation, the surface being freshly excited each time. Three different series, each corresponding to a different intensity, showed a direct relation between intensity of red radiation and extent of flashing-up. With small intensity the flash is small, with great intensity it is marked. With the 100 c.p. lamp at about eight inches from the surface a red flash of one second produces an observable flashing-up of phosphorescence as

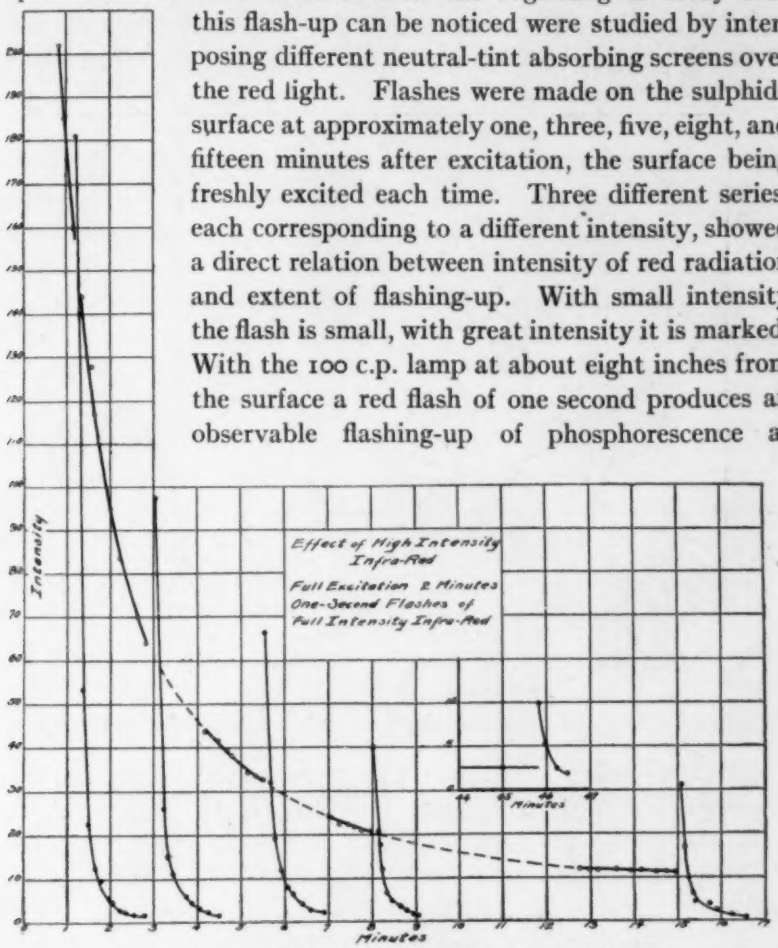


FIG. 3.—Effect of high intensity of infra-red

close as thirty seconds from the beginning of decay, while at 15 minutes the flash is several hundred per cent of the brightness of phosphorescence just preceding it. With a tenth of this intensity of the red the flashing-up is much smaller, and not noticeable till a

later time. Fig. 3 gives a typical decay-curve with the effects of a red flash of high intensity at various times. In the observations plotted in Fig. 4 a large range of intensities of red light were used and the flashes were all made at approximately five minutes after excitation. They show the dependence of the amount of the flashing-up on the intensity of red light. These have been plotted diagrammatically to one side.

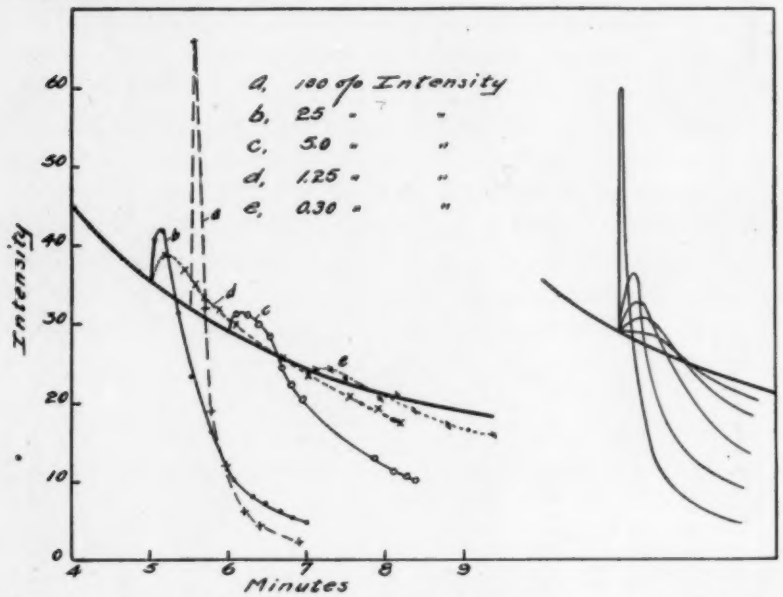


FIG. 4.—Effect on flashing-up phenomenon of varying the intensity of infra-red

Effect of weaker excitation.—Fig. 5 shows three decay-curves, corresponding to three different intensities of the exciting light. At approximately the same times after the beginning of decay the same exposure to red light has been given to each. It is at once evident that the significant factor is not the brightness to which the phosphorescence has fallen but the time that has elapsed since excitation. The first red flash on the lowest curve, for instance, occurs when the brightness of phosphorescence is already lower than it is on the highest curve at a time when the flashing-up is very marked.[1]

[1] The indicated flash-ups on the lowest curve (*c*) were observed qualitatively, but no measured points above the curve were obtained.

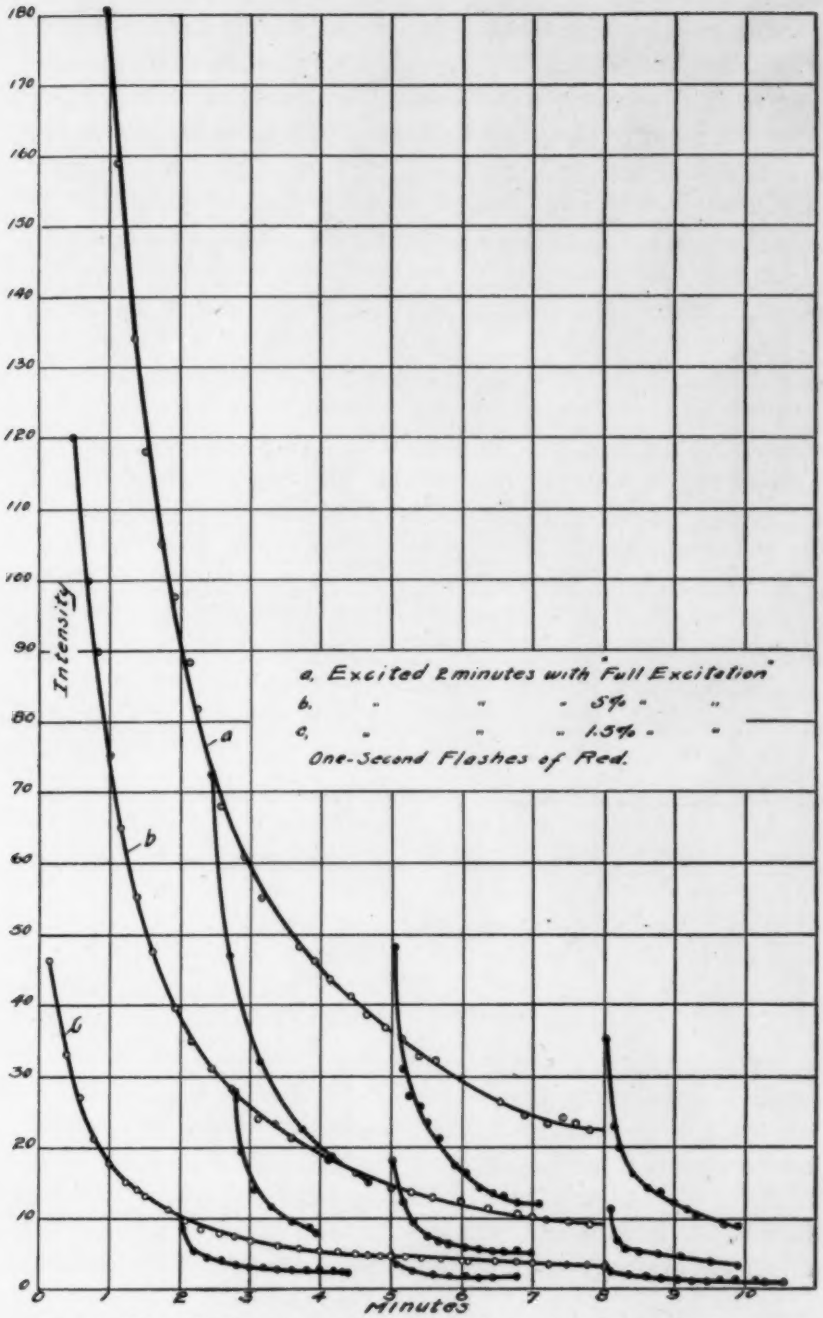


FIG. 5.—Effect of different intensities of excitation on the flashing-up phenomenon

Effect of shorter excitation.—Exactly similar results to those of Fig. 5 are obtained if lower decay-curves are produced by shortening the excitation to a few seconds. These confirm the conclusion that the occurrence of the flashing-up is determined not by the

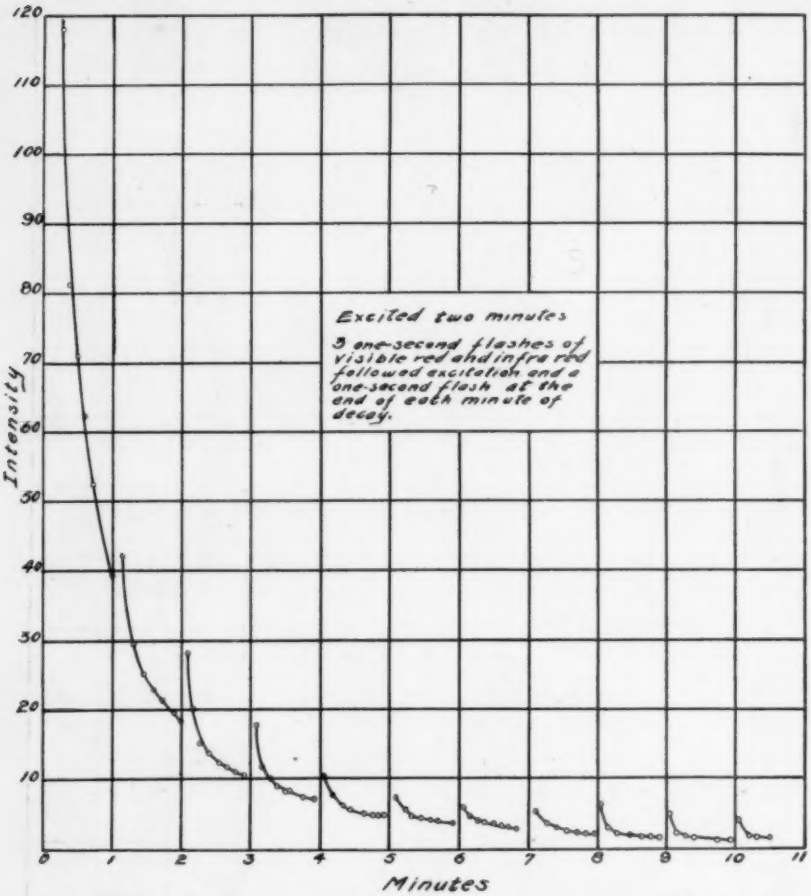


FIG. 6.—Effect of previous exposure to red light on the susceptibility to flashing-up

condition of phosphorescence as measured by its brightness, but by the time that has elapsed since the beginning of decay.

Effect of previous exposure to red or infra-red.—Short exposure to long-wave radiation, which in general acts to decrease the brightness of phosphorescence, does, unlike decreased or shortened

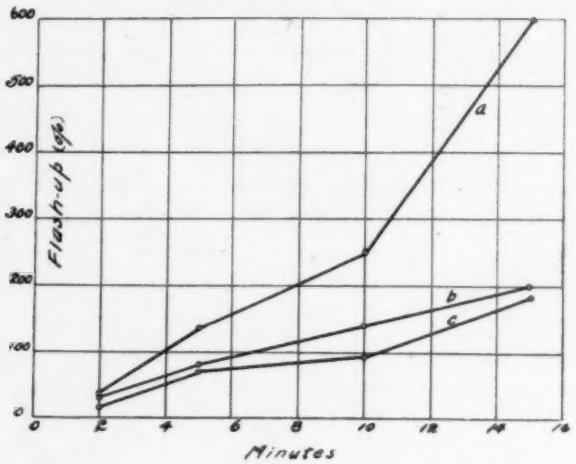
excitation, hasten the condition of flashing-up. An exposure of 3 seconds to red light, immediately after excitation, brings the substance to a condition such that there is a pronounced flashing-up long before flashing-up would occur under other circumstances when flashed with a given intensity of red light. Successive exposures to red light produce strong flashes of phosphorescence apparently for an indefinite period. We have watched and measured these bursts of phosphorescence produced by successive flashes of one second each at intervals of one minute up to the number of ten, when the phenomenon was as pronounced as at the start (Fig. 6).

Observations on different samples.—The zinc sulphide used in these experiments was a Verneuil preparation furnished by Eimer & Amend. With the possibility in mind that it was of a quite different character from that employed by previous observers an effort was made to secure other specimens. Through the kindness of Professor E. L. Nichols we were provided with a piece of the sidot-blende screen used by him and Professor Merritt in their work, and with a specimen of "Emanationspulver." Some of the experiments above described were repeated with these samples. With all three specimens the flashing-up phenomenon is present, but in different degree; similarly other properties, such as color of phosphorescence and shape of the decay-curve, are different, and it will be seen that they fall into a certain uniform order with respect to all their properties.

Flashing-up phenomenon.—The flash-up is least marked in the sidot-blende screen, most marked in the "Emanationspulver," our sample of zinc sulphide lying between. Fig. 7 shows the percentage increases in brightness due to exposure to infra-red radiation, of uniform intensity in all cases. The infra-red was allowed to continue its action when once turned on, the luminosity being followed by the Brodhun sector till it reached a maximum. The sidot-blende screen, which shows the least flashing-up, also shows the most rapid decay under the infra-red.

Decay-curves.—The decay of phosphorescence is frequently plotted with time as abscissas, and $\frac{1}{\sqrt{I}}$, where I is intensity, as ordinates. A close approximation to a straight line has usually

been found, this corresponding to a decay law of $I=\frac{1}{(a+bt)^m}$ where $m=2$. Nichols and Merritt find two straight lines, merging one into the other a few seconds after the beginning of the decay. Werner, in agreement with the theory of Lenard, finds two processes in the decay: one a long process which plots as a straight line in terms of $\frac{1}{\sqrt{I}}$ against time; the other a short process superposed on the long, and following an exponential law. In sulphides contain-



a. Emanations Pulver
b. Zinc Sulphide
c. Sidot Blende

FIG. 7.—Magnitude of flashing-up for different samples. Time is that elapsed since excitation.

ing a large amount of active metal the short process lasts for a much shorter period than in sulphides containing a small quantity. Consequently the decay-curves when plotted in terms of $\frac{1}{\sqrt{I}}$ more quickly become straight lines in the former than in the latter case. Werner has also experimentally determined the constants in the two equations $I_1=\frac{1}{(a+bt)^2}$ and $I_2=ae^{-\beta t^m}$ such that the summation of these two curves represents very satisfactorily the different decay-curves obtained with different compositions of the sulphide. Upon plotting the decay-curves of our sample of zinc sulphide in

terms of $\frac{1}{\sqrt{I}}$ a curve concave to the time-axis was obtained, which approached nearer a straight line as time went on, although even at thirty minutes it is questionable whether perfect straightness is reached. A decay-curve of this substance—the mean of many

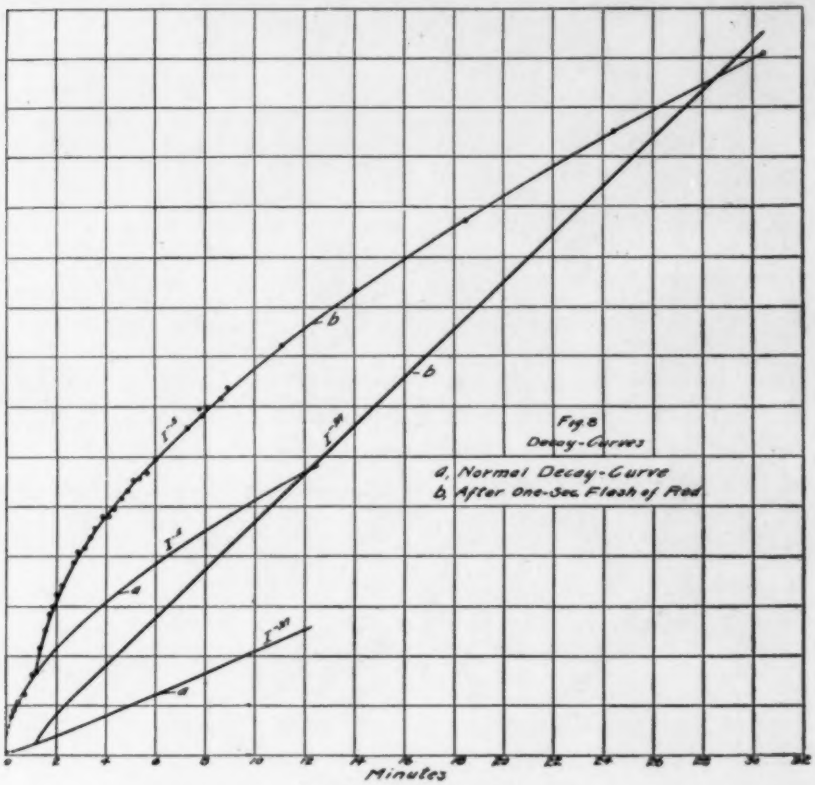


FIG. 8.—Decay-curves

observations—plotted in terms of $\frac{1}{\sqrt{I}}$ is shown in Fig. 8. There is also shown a curve obtained from the substance after one second's exposure to red light. This gives a similar concave curve, not, as has previously been supposed, the first curve with its origin shifted. It is a matter of some interest to note that all our curves plot as straight lines in terms of $\frac{1}{I^{.97}}$, a fact learned upon plotting

them on logarithmic paper. Great significance is not attached to this point because, for reasons below noted, it appears more probable that we have, as Lenard and Werner have proposed, two superposed processes, following different laws. That the resultant curves plot as straight lines in terms of $\frac{1}{I^{.97}}$ is perhaps only an accident.

The decay-curves of the three different samples at our disposal show their different degrees of straightness in exactly the order of their exhibition of the flashing-up effect. The sidot-blende screen shows the greatest curvature, while the "Emanationspulver" is, within the error of observation, a straight line from one minute onward. These are shown in Fig. 9. If we accept the two processes of Lenard, and consider these

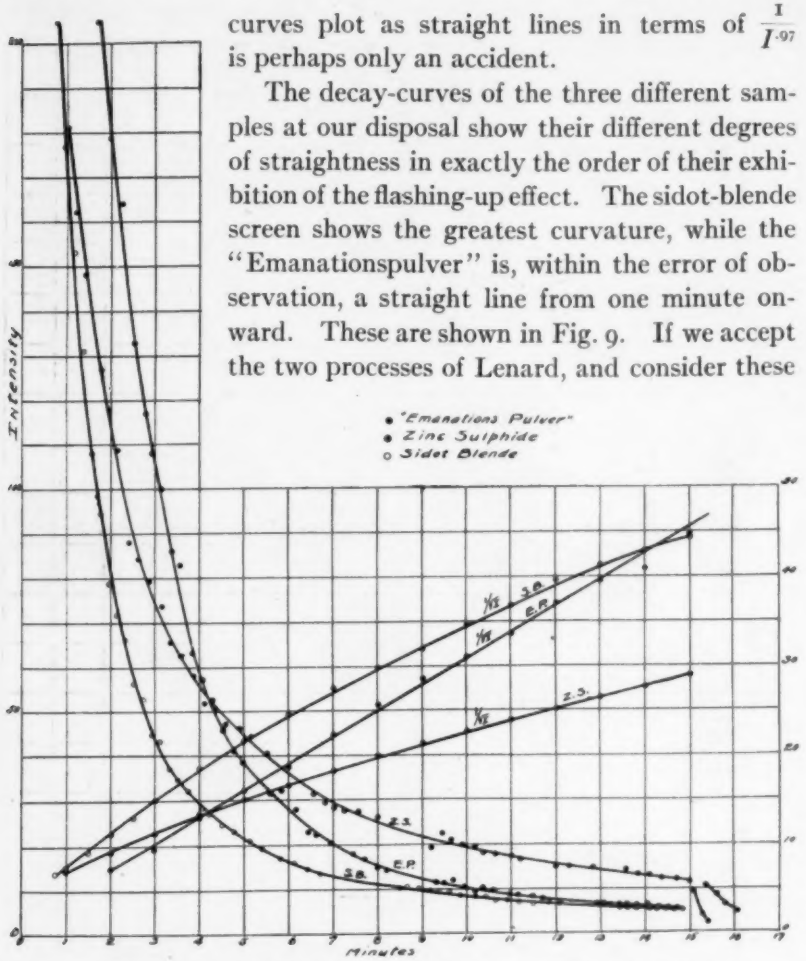


FIG. 9.—Normal decay-curves of three specimens of zinc sulphide

three samples as sulphides of different content of active metal, exhibiting the two processes in different degree, in agreement with the work of Werner, then it appears that the flashing-up

phenomenon is a characteristic of the long process (*Dauerprozess*) which plots as a straight line, while the rapid drop is a characteristic of the short process (*Momentanprozess*), which furnishes the initial curvature (when plotted in terms of $\frac{1}{\sqrt{I}}$ against time.)

SPECTROSCOPIC WORK

Following these photometric observations attention was turned to spectroscopic lines of investigation. The dependence of the flashing-up effect on the wave-length of the exciting light and of the extinguishing light was first investigated. Then with the assistance of photography the spectral character of the phosphorescent light was studied.

For the first problems a small Fuess quartz spectrograph was employed. Owing to the extreme obliquity of the plate with respect to the axis of the instrument it was possible to cut an opening in the side of the camera, through which the phosphorescent surface could be observed during excitation or decay and through which red radiation could fall upon the sulphide when desired. An iron arc was used in studying the characteristics of the substance under different kinds of excitation. Zinc sulphide has a maximum of response to equal energy excitation in the blue and the violet, falling off to zero in the green, and being of lower value in the ultra-violet. Study of the effects of the flashes of red upon the strip of phosphorescence caused by the iron arc spectrum disclosed no noticeable difference at different wave-lengths. No matter by what light it was excited, the phosphorescence showed, after proper lapse of time, this flashing-up of brightness.

A Nernst glower (focused on the slit by a glass lens) was used in studying the effect of different wave-lengths of extinguishing light. The phosphorescent surface was of such size that it could be moved past the slot in the camera back, and six observations could be made, at appropriate intervals after excitation. There are two maxima of extinguishing action, one at $0.7\ \mu$, the other at $1.3\ \mu$ (for a normal spectrum), with a minimum between, at $1.1\ \mu$. The action extends to about $1.6\ \mu$ and to about $0.6\ \mu$. A rather interesting phenomenon was noted in a series of observations where

the exposure to extinguishing light consisted of flashes of equal length. This was that the flashing-up effect is first apparent on the short-wave side of the extinguishing spectrum, afterward progressing toward the long-wave side. At first it was thought that this must be actual production of phosphorescence, i.e., excitation; but upon cutting off all the blue and blue-green light by a yellow stained film the effect persisted, and finally, upon exposing a surface

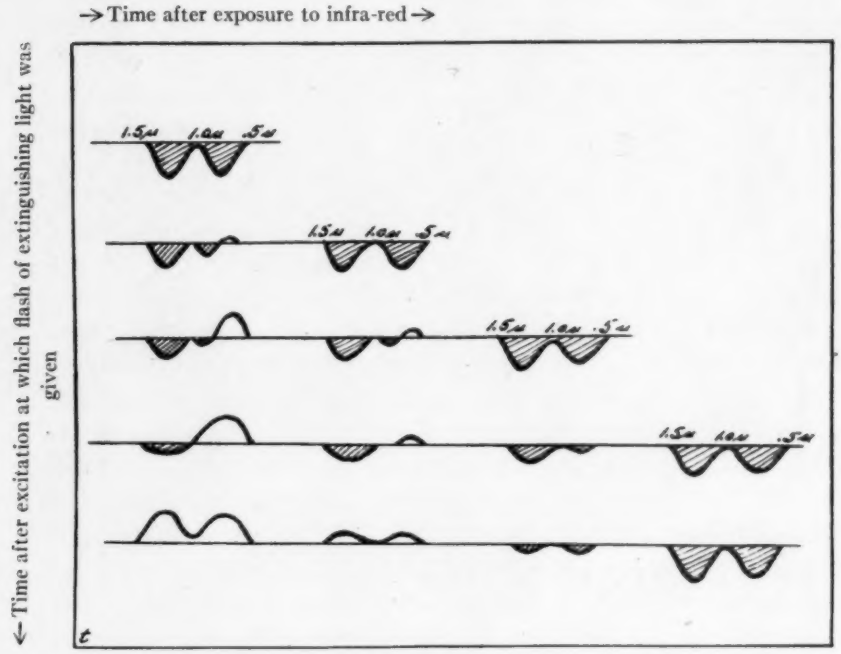

→Time after exposure to infra-red→
←Time after excitation at which flash of extinguishing light was given
1.5μ 1.0μ .5μ
1.5μ 1.0μ .5μ
1.5μ 1.0μ .5μ
1.5μ 1.0μ .5μ
t

FIG. 10.—Progress of flash-up from green to infra-red

thoroughly extinguished beforehand by red light and by rest in the dark, no trace of luminescence was produced, clearly showing this to be not an excitation of phosphorescence, but a flashing-up of decay.

Fig. 10 shows diagrammatically the apparent course of the extinguishing action. From top to bottom the time after excitation at which the red flash occurs is supposed to increase, as shown; from left to right is illustrated the history of the decay after the

flash of red. Elevation above the reference line indicates the exposed area to be brighter than the surrounding surface, while depression below the line indicates the exposed area to be darker. The scale of wave-length refers to the extinguishing light. These experiments were made only with our own specimen of zinc sulphide, as the quantity of the other samples did not make experiment with them feasible.

Spectral character of the phosphorescent light.—Visual observation shows the three samples of zinc sulphide to be different in the color of their phosphorescence, differing in the same order as that previously noted in respect to the extent of flashing-up and

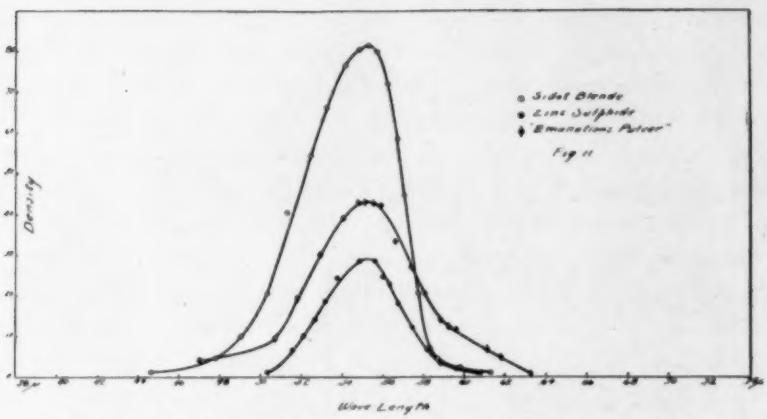


FIG. 11.—Densities of spectrograms of three specimens of zinc sulphide

the shape of decay-curves. The "Emanationspulver" is the most yellow, the sidot-blende screen the most blue-green. This difference shows in the fluorescence, and lent itself readily to photographing with a grating spectrograph of large aperture giving a spectrum 75 mm in length from 0.7 μ to 0.4 μ. The blue and violet mercury lines from a Cooper-Hewitt lamp served as excitation, and a screen of potassium bichromate reduced the intensity of the reflected blue light so that it was not troublesome in the spectrograph. The three negatives obtained, with a narrow slit and exposures of 9 to 24 hours, show no evidence of structure and in spite of the observed difference in color appear rather nearly

alike. In Fig. 11 are plotted the densities of the negatives against wave-length. Cramer "Spectrum" plates were used. One of the most important points to notice is that the apparent color-differences are due to but slight differences in the spectra, owing to the fact that the radiation lies in the yellow-green part of the spectrum where hue changes rapidly with wave-length. As a consequence a transformation from one form to another during decay, after color is no longer observable, would be difficult to pick up without considerable dispersion.

Photography of the spectrum of the phosphorescence.—A very difficult photographic problem arises when an attempt is made to photograph the spectrum of phosphorescence, because of the excessive faintness of the phenomenon. Pierce, who as yet has done practically the only work of this kind, has not carried it beyond two minutes after excitation. For the study of the flashing-up it is necessary to photograph at least five minutes after excitation—a fact which has necessitated special apparatus and exposures of excessive length.

The most interesting problems are to determine whether any change occurs in the spectrum when the phosphorescence flashes up and whether the spectral character undergoes a change during decay into a form more susceptible to flashing-up. In the case of calcium sulphide, Stokes noted a distinct difference of color in the flash-up as compared with the unflashed phosphorescence. This would suggest the possibility of a different band of light or of only one side of the original band being responsible for this effect. Nichols and Merritt have observed the blue side of the zinc sulphide band to increase in brightness under infra-red radiation while the rest of the band decreased.

In order to search for effects of this kind at late periods in the decay a large photographic phosphoroscope was constructed. Shown in plan in Fig. 12, it consists essentially of a flat disk of metal 1 foot (30 cm) in diameter mounted on the minute shank of a strong clock. Near the rim of the wheel is a shallow groove, filled with zinc sulphide powder. Through A comes the exciting light (blue lines of a Cooper-Hewitt arc), to be reflected downward by a mirror, M, over the groove. Through B comes strong red light to

bring the sulphide to a standard condition before excitation. At C is a Nernst glower, inclosed in a light-tight box. An image of the glower is formed by the lens, D, through the CS_2 and iodine cell, E, and strikes the phosphoroscope wheel after reflection from the mirror M_3. The Fuess quartz spectrograph, F, points directly down upon the disk, and a lens forms upon its slit an image of the phosphorescent surface and of the Nernst image when desired.

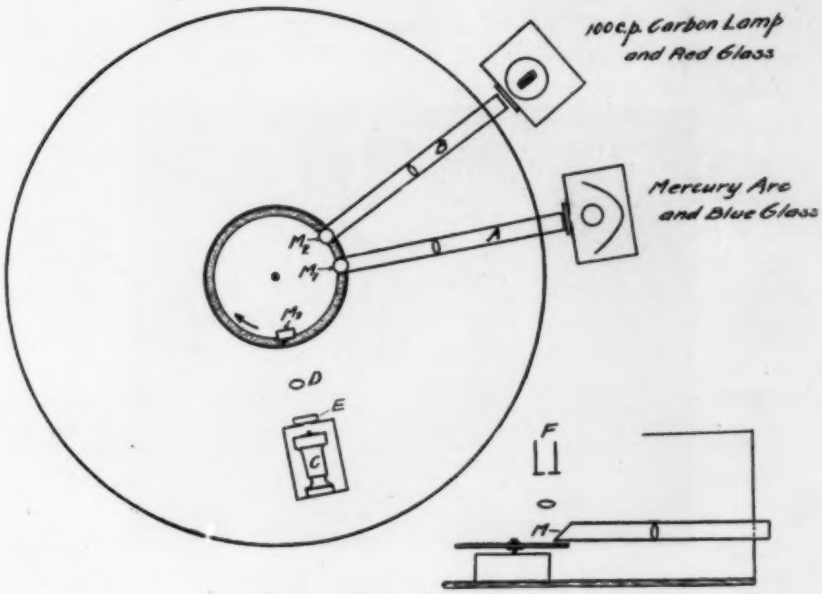


FIG. 12.—Large phosphoroscope

With this apparatus four photographs were obtained, a pair corresponding to one minute after excitation, and a pair corresponding to fifteen minutes after excitation. The exposures were from fifty hours for the shortest at one minute, to ten days for the longest at fifteen minutes. In one of each of these the extinguishing image of the Nernst glower was focused to cover half the groove.

No shift whatever in the position of the band of phosphorescence nor change in its character can be detected on comparison of the one minute and fifteen minute positions. Owing to the small

dispersion, however, it is just possible that as much change as that between the sidot-blende and the "Emanationspulver" might be present (Fig. 13).

The exposures at 15 minutes under the infra-red radiation showed beyond question that the total light flashes up; that is, it is not caused by a new band or by a marked increase in one part of the yellow-green band. On the most exposed fifteen minute plate a faint additional band was present on the blue side,

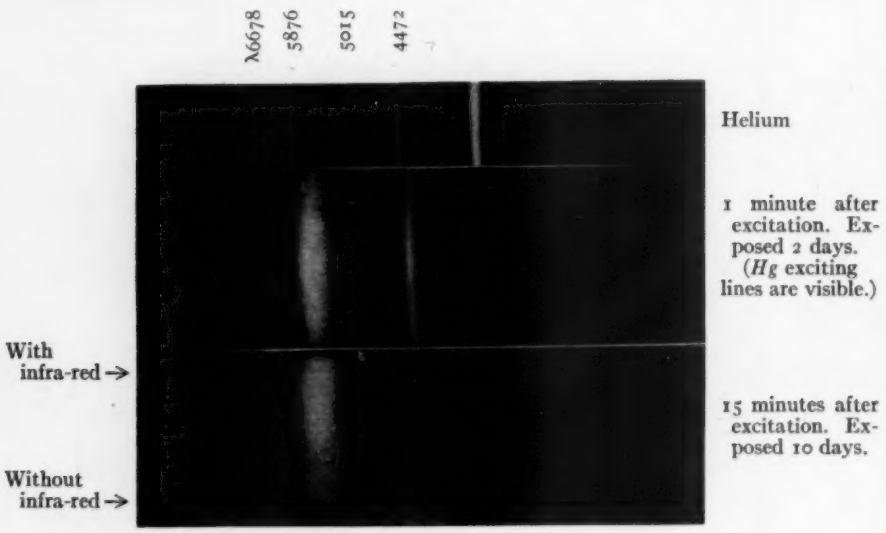


FIG. 13.—Spectra of phosphorescence (enlarged 3 times)

which could not be traced to scattered light or other false cause. (This band was too faint to reproduce in Fig. 13.) In position it corresponds to the band in the blue of very short duration which has been previously observed, notably by Nichols and Merritt. The exposure at one minute under infra-red showed no blue band nor difference in character of the main band.

It is apparent from these results that the phenomenon of flashing-up will find its most probable explanation in some process affecting the total emitted light, the process varying in some manner during decay, rather than in any outburst of different spectral constitution.

THEORETICAL

The flashing-up phenomenon has not received great attention in existing theories of phosphorescence, which have had sufficient task before them to explain the more prominent phenomena of excitation and decay. What follows is a suggestion as to how the experiments just described may be explained in general accordance with the ideas of Lenard.

Lenard (*op. cit.*) classes zinc sulphide among the phosphorescent alkaline earth sulphides, which consist of an "active metal," an "alkaline earth sulphide," and a "flux." In zinc sulphide the zinc may be both the active metal and the metal of the sulphide, or other metals such as strontium, copper or manganese in minute quantities may constitute the active metal. The action of light is the photoelectric effect, which tears out the photoelectric electrons. In their oscillatory return these, by resonance, set in motion the emitting electrons, whose vibrations constitute the fluorescent and phosphorescent light. Certain of the former electrons are captured or stored up by the sulphur atoms, while others are dispersed through the surrounding atoms without becoming attached. The return of the stored-up electrons due to collisions or approaches of the metal atom and the loaded sulphur atom, will cause luminescence following the law $I=\frac{1}{(a+bt)^2}$; the return of the unattached electrons will follow the law $I=ae^{-\beta t^m}$. In accordance with this hypothesis Werner considers the sulphides with a large quantity of active metal, those which show a straight-line decay (plotted as $\frac{1}{\sqrt{I}}$ against time), as having but a small number of unattached electrons after excitation, and vice versa.

The effect of elevated temperature and of infra-red radiation is held to be a shaking-up of the atoms, by which the frequency of the collisions is increased. Red and infra-red produce the agitation by resonance, setting in motion the larger aggregates which consist of sulphur atoms and attached electrons. The size and free period of these aggregates are large; they therefore respond to long waves, just as the small photoelectric electrons respond to short-wave excitation.

It will be seen that this theory calls for no direct effect of red radiation on the unattached electrons: in other words, only that period of the decay which is wholly represented by the straight-line relation is considered. In this region an accelerated rate of return of electrons is to be expected. This will cause an initial flash followed by a rapid decay, precisely what does occur.

Let us now carry this reasoning of Lenard's further and apply it to the unattached electrons to which is due the initial curvature of the decay when plotted in the manner we have considered. The condition we imagine is that in returning to the metal atom these have to pass between the sulphur atoms, loaded and unloaded. If the sulphur atoms are set into vibration by the incidence of long-wave energy it is evident that the chance for an unattached electron to reach its goal is much reduced, for it stands a big chance of being captured by collision with the rapidly moving sulphur atoms or aggregates. We might then expect that, while the luminescence due to the return of the originally captured electrons will flash up and then decrease, that due to the unattached electrons will suffer an immediate decrease owing to their capture in a manner not productive of light. The magnitude of this latter drop in brightness may be sufficient to mask the initial flash due to the freeing of the attached electrons.

One consequence of this train of events would be that a short flash of infra-red energy would have a different effect on the subsequent decay, depending on whether it occurred during the early part of decay or at a later period. Let us assume that the effect of a given amount of long-wave energy on the aggregates composed of sulphur atoms and attached electrons is the breaking down of a certain fraction of them by collision. Let the brightness of the phosphorescence at any time be a function of the number of such aggregates existing and subject to the normal collisions at the working temperature. Then if an early flash of red both breaks down a fraction of the total aggregates and builds up new ones as indicated above, while later it breaks down the same fraction but builds up comparatively few new ones, then the brightness of the sample submitted to the earlier flash will at any time subsequent to both flashes be the greater. In short, the degree of extinction of phosphorescence

produced by the same quantity of long-wave energy should increase with the increase of time after excitation.

This is actually what does take place. In a previous paper[1] it was shown by photographs of a phosphorescent zinc sulphide surface which had been given a series of exposures to red light that greater extinction occurred as time went on. This has been checked

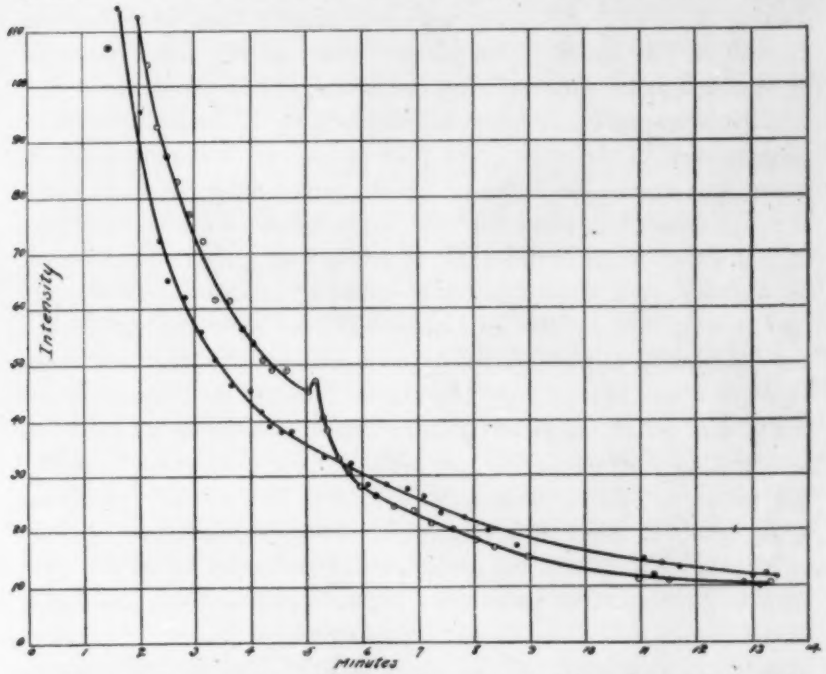


FIG. 14.—Decay produced by identical exposure to infra-red at 1 minute and 5 minutes after excitation.

by a series of equal exposures upon a phosphorescent surface in the quartz spectrograph using the light of a Nernst glower. A photographic print from this showed clearly the continued increase of extinguishing effect with time up to about two hours, after which it became constant.[2] So marked was this effect that it was thought

[1] "Further Studies of the Fire-Fly," *Physical Review*, **31**, 637, 1910.

[2] If the exposure to the photographic plate is made too soon after the last exposure to infra-red (say within 1 or 2 hours) the appearance is as though the extinguishing effect were decreasing again. This is due to the flashing-up phenomenon, which after this time becomes very large and long continued.

well to look for it by visual observation with the photometric apparatus. The result is shown in Fig. 14. The phosphorescent surface was excited, half of it covered with a metal plate, and the other half given, after 12 seconds, a 1-second flash of infra-red. The decay of both halves was followed to five minutes, then the first half was covered and the other half given an exactly similar flash of infra-red. After this measurements of each were made as plotted in the figure. The greater effect of the same quantity of infra-red at the later point in the decay is clearly indicated.

This proposed explanation would place the three samples which were studied in the same order with respect to character of decay curve and extent of flash-up, which agrees with the facts. No attempt is made to incorporate an explanation of the slight difference in color of the samples, for at present we have no data to tell us whether their constituents are exactly the same. The difference in color may be due to a cause different from that producing the differences in the decay-curves. The latter, from the work of Werner, are known to be producible by different proportions of the typical constituents and a difference of that character has been assumed as responsible for the difference in the decay-curves of our samples. Other differences may exist, and until we have an opportunity to make up for ourselves specimens of these sulphides of known composition, we cannot follow out with profit the connection between the flashing-up phenomenon and the character of the decay-curve more clearly than has been done.

The phenomena of the progress through the spectrum of the flashing-up effect as shown in Fig. 10 may be explained in agreement with the preceding paragraphs by assuming the atoms or aggregates which respond to long-wave energy as of different sizes, but all very much larger than the unattached electrons. In passing through a group of vibrating atoms the probability of the electron being captured is less the smaller the atoms. The smaller atoms are those thrown into vibration by the shorter wave-lengths. These, then, would be the first to show a decrease of the effect due to capturing unattached electrons; that is, they would be the first to show the flashing-up.

A somewhat simpler explanation is, however, possible, and that

is that we here have differences due to different quantity of extinguishing radiation. In the Nernst glower the intensity of the radiation increases rapidly toward the infra-red. The study of the effect of different quantities of long-wave energy, plotted in Fig. 4, shows that while a small quantity produces a small flash-up, the flash is shown to die down. As the experimental difficulties were large in the observation of Fig. 10 it is possible that the effect was first noticed on the short-wave side because it had not disappeared too quickly to be caught. More exact quantitative measurements are here necessary.

SUMMARY AND CONCLUSION

It has been found, contrary to general belief, that phosphorescent zinc sulphide exhibits a flashing-up of luminosity under the incidence of long-wave energy. The peculiarity of this effect is that it becomes noticeable only after the decay of phosphorescence has proceeded for some time. Its occurrence is dependent on the time elapsing since excitation, for neither less intense excitation nor shorter excitation will hasten the bringing of the phosphorescent substance to the proper condition. Short exposure to long-wave energy does, however, hasten the attainment of the proper condition for showing the flash-up. The magnitude of the flash-up varies directly as the intensity of the red or infra-red radiation.

Study of several samples indicates that there is a direct relationship between the character of the decay-curve and the magnitude of the flashing-up phenomenon. The samples whose decay plots nearest a straight line in terms of $\frac{1}{\sqrt{I}}$ against time, show the effect more than the ones whose decay plots as a curve.

Spectroscopic observation indicates no qualitative difference in the phenomenon due to differences in the wave-length of the exciting light. The flashing-up effect appears to occur the sooner, the shorter the wave-length of the extinguishing light. Spectrograms show no change in the spectrum of the phosphorescent light during decay. The spectrum of the flash is the same as before the flash, indicating that the phenomenon is not one of a separate band or part of the radiation, but a phenomenon of the total emission.

An attempt is made to develop a theory of the effect, based on the theory of Lenard. The flashing-up is ascribed to the stored-up electrons, which determine the shape of the decay-curve in the later part of decay; the quick drop in luminosity in the early part of decay is ascribed to the unattached electrons, which influence the initial direction of the decay-curve.

It is our intention to follow up this work with similar experiments on freshly prepared sulphides of known composition, to extend the work to other substances, and to study the similar phenomena caused by heat.

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THE SUN'S ENERGY-SPECTRUM AND TEMPERATURE[1]

By C. G. ABBOT

Spectro-bolometric determinations of the solar constant of radiation are attended by a number of by-products, among them the distribution of energy in the sun's spectrum at the earth's surface, and also outside the atmosphere. In the earlier solar-constant observations of the Smithsonian Astrophysical Observatory, the form of the energy-curve outside the atmosphere was investigated rather frequently, involving independent determinations of the transmission of the apparatus. But it soon proved that the value of the solar constant was only very slightly affected by wide changes in the supposed form of the extra-atmospheric energy-curve,[2] and we have generally assumed the form as known, and determine the transmission of the optical system therefrom. However, in each year of observation several complete determinations have been made, and the conditions of observation have varied so much that it seems hardly possible that the mean value to be given below can be very far astray, notwithstanding the great difficulty of the measurements, and the considerable divergence of the several determinations.

In short, the determinations have been made as shown in Table I.

The initials of observers refer to C. G. Abbot, F. E. Fowle, L. R. Ingersoll, L. B. Aldrich, and W. S. Adams. I take this opportunity to express my obligation to the four last-named gentlemen and to Miss Graves for their aid in the work. A statement of the method usually adopted for the observation may be found in Vol. II of the *Annals of the Astrophysical Observatory* (see pp. 24 and 50 to 57). At each of a large number of wave-lengths in the solar spectrum it is required to determine: (1) the intensity at the bolometer; (2) the selective transmission of the spectroscope; (3) the selective reflection of the heliostat;

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] See Abbot and Fowle, *Astrophysical Journal*, **29**, 280, 1909.

TABLE I

Conditions of the Determinations

Observers	Date	Station	Altitude Meters	Heliostat	Prism	All Mirrors		Collimation
						No.	Kind	
C.G.A..... F.E.F.....	1903 Feb.–Apr.	Washington	10	One mirror Siderostat	Flint glass	4	Silvered glass	Used
C.G.A..... L.R.I.....	1905 June–Oct.	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A..... L.R.I.....	1906 August	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A..... L.B.A..... W.S.A.....	1908 Sept.–Oct.	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A..... L.R.I.....	1909 June–July	Mount Wilson	1730	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A..... L.R.I.....	1909 August	Mount Wilson	1730	Two mirrors Coelostat	U.V. crown glass	4	Silvered glass	None
C.G.A..... L.R.I.....	1909 July–Aug.	Mount Wilson	1730	Two mirrors Coelostat	Quartz	4	2 Silvered glass 2 Magnalium	None
C.G.A.....	1909 September	Mount Whitney	4420	None	Quartz	2	Magnalium	None
C.G.A..... F.E.F.....	1910 July	Mount Wilson	1730	Two mirrors Coelostat	Quartz	4	2 Silvered glass 2 Magnalium	None
C.G.A.....	1910 August	Mount Whitney	4420	None	Quartz	2	Magnalium	None

(4) the transmission of the atmosphere. A bolograph indicates the first, and measurements on a series of bolographs, taken at different zenith distances of the sun, furnish the means of computing the last. The reflection of the heliostat is determined by taking bolographs: (*a*) with the ordinary mirror or pair of mirrors; (*b*) with a substitute mirror or pair of mirrors; (*c*) with the combination of both regular and substitute mirrors. The relative transmission of the spectroscope is determined by first passing the rays through an auxiliary spectroscope, selecting certain wavelengths and observing their intensity: (*d*) as transmitted by the auxiliary spectroscope; (*e*) as transmitted by both spectroscopes.

The observation (*d*) is made by removing the bolometer to occupy the position usually occupied by the slit of the usual spectro scope. When transmitted by both spectroscopes, the beam becomes nearly monochromatic, but its intensity should be observed by measuring the area included in a bolograph taken by moving this monochromatic image over the bolometer, not (as stated at p. 51 of the *Annals*) by observing the maximum deflection produced by the monochromatic image. Under certain circumstances, however, the areas and the maximum deflections are nearly proportional. In all the earliest observations at Washington this seemed to be the case, and also at Mount Wilson in 1905–1906. Though we registered the type (*e*) bolographs in 1905–1906 at Mount Wilson, we generally measured merely the maximum ordinates in the reductions as published at pp. 51 and 105 of *Annals*, Vol. II. In order to see what errors may have resulted, the *areas* have lately been measured for all the determinations given in the *Annals*. They prove to be so nearly proportional to the maximum deflections that no error as great as the uncertainty of measurement is found to be indicated. In the year 1908 several determinations were made with another prism of different dispersion in use in the auxiliary spectroscope, and unfortunately the areas were not even observed. As some determinations made with still another prism, in 1909, which were reduced by both methods, indicate large errors in the deflection method, it is thought best to reject entirely all the determinations (those of 1908) in which the areas are not available.

In order to make the several determinations comparable, they are first reduced from the prismatic to the normal, or wave-length scale. In order to do this the deviation of the prism is platted against the corresponding wave-length, and the tangents to the resulting curve are determined at the various prismatic deviations. In order to reduce the several determinations to comparable intensity-scales, a multiplying factor for correcting each curve has been determined, in such a manner as to make all the curves agree as well as possible between wave-lengths 0.5 μ and 1.2 μ. For the sake of brevity not all the determinations are here given separately, but all determinations made with the same arrangement of apparatus, and on the same approximate date, are combined, and the mean of several such determinations is here used. It should be remarked that the Mount Whitney determinations of 1909 depend on spectroscopic transmission factors determined at Mount Wilson in 1909, and the Mount Whitney determinations of 1910 similarly depend on Mount Wilson spectroscopic factors of 1910. Apart from this dependency, all determinations here given are wholly independent.

The accuracy of the determinations at Mount Wilson in 1905 and 1906 is prejudiced by the fact that the scale of the galvanometer had not at that time been investigated. But in the mean of so many determinations as are included in the 1905–1906 group, a great many different parts of the galvanometer scale were used, and it is probable that the scale-error is principally eliminated except in the ultra-violet, where the deflections were all near zero of the scale, and disproportionately large. This part of those curves is rejected in deriving the mean values in the following discussion. The galvanometer scale was strictly uniform in all the subsequent measurements, excepting those on Mount Whitney in 1910. The deviation from uniformity for the 1910 Mount Whitney measurements was investigated and found negligible in comparison with other sources of error.

In all the determinations the accuracy falls off rapidly in the ultra-violet spectrum. With the flint-glass prism the dispersion is so great, and the deflections and areas recorded in determining spectroscopic transmission consequently so small, that such values have not been used in fixing the mean value to be given

for wave-lengths less than 0.42 μ. Only one complete determination has been made thus far with the ultra-violet prism. Although it is somewhat abnormal it is not rejected. Determinations made with the quartz prism have fair weight to about wave-length 0.37 μ. Beyond this they depend upon more and more doubtful data for the transmission of the spectroscope; and beyond wave-length 0.35 μ they depend wholly on extrapolation for this quantity.

Table II is a summary of the results. It should be considered in connection with the table given above. In the first column are wave-lengths, next mean intensities in the solar spectrum outside the atmosphere (in arbitrary units), and in the remaining columns the departures from the mean intensity in percentages for the average curves determined on the dates given at the bottom of the table.

I do not venture to decide whether the departures from the mean may be attributed in part to real changes in the distribution of energy in the solar spectrum, or are altogether the results of error in the difficult determination of spectroscopic transmission. The argument on that subject remains as yet where it was left by Abbot and Fowle.[1] It is very pleasing to note that the mean Washington, Mount Wilson, and Mount Whitney determinations agree with one another as well as the separate determinations at the three places agree each with each.

For the sake of those who may desire to compute the distribution of energy in the solar spectrum at the earth's surface, I give also the transmission of the atmosphere at various wave-lengths. The values represent the fraction of the intensity of the solar beam outside the atmosphere which would remain in a direct beam transmitted vertically to the earth's surface. Average values for cloudless (but not necessarily hazeless days) are given. as they have been found at Washington, Mount Wilson,[2] and Mount Whitney, for the various wave-lengths. To compute the

[1] *Astrophysical Journal*, **29**, 287, 1909.

[2] For the reason stated in this Journal, **33**, 193, 1911, the Mount Wilson transmission coefficients published in Vol. II of the *Annals of the Astrophysical Observatory* are about 1.4 per cent too low.

TABLE II

Solar Energy-Spectrum Determinations

Wave-Length	Mean Intensity	Individual Groups of Determinations. Percentage Deviations from Mean*										Average	Percentage Probable Error
		1	2	3	4	5	6	7	8	9	10		
0.30.....	(440)											—	—
0.325.....	1285								−31.	+67.	+86.	61.	30.
0.35.....	2700				+51.	−30.	−17.	−22.	−12.	+16.	+10.	23.	7.3
0.375.....	3464				+29.	−11.	− 4.4	− 9.4	− 4.6	+ 3.0	− 1.9	9.0	2.7
0.39.....	3620			− 0.6	+15.	−12.	− 2.2	− 6.1	− 0.6	+10.	− 1.9	6.0	1.7
0.42.....	5261	+ 0.0	+6.9	−17.	− 4.7	− 3.5	+ 4.4	+ 2.2	+ 3.4	+ 7.5	+ 0.2	5.0	1.3
0.43.....	5340	+ 2.4	+6.4	−18.	−10.	− 4.5	+ 3.4	+ 1.2	+ 5.1	+11.	+ 3.1	6.5	1.7
0.45.....	6047	− 2.7	+1.1	−15.	− 8.7	+ 0.0	+ 8.4	+ 5.4	+ 5.3	+ 7.6	− 1.1	5.5	1.4
0.47.....	6253	− 4.7	−2.2	−16.	−12.	+ 4.9	+ 7.8	+ 5.8	+ 1.9	+12.	+ 2.3	7.0	1.8
0.50.....	6064	+ 0.3	−7.6	−17.	−13.	+ 3.5	+ 9.9	+ 7.1	+ 5.2	+12.	+ 0.2	7.6	1.9
0.55.....	5627	− 7.2	−7.8	−15.	−12.	+ 5.0	+ 8.5	+ 7.2	+ 9.4	+11.	+ 0.9	8.4	2.1
0.60.....	5047	−11.	−5.3	−14.	−10.	+ 7.4	+ 7.9	+ 7.3	+ 8.0	+ 9.4	+ 1.1	8.1	2.1
0.70.....	3650	− 4.4	−0.1	− 2.1	0	+ 1.2	+ 1.0	− 1.6	+ 1.9	+ 4.4	− 0.5	1.7	0.4
0.80.....	2672	+ 4.2	+4.8	+ 6.7	+ 5.4	− 4.3	− 4.2	− 5.9		− 4.1	− 1.9	4.6	1.2
1.00.....	1664	+ 4.4	+1.6	+ 6.4	− 1.0	− 1.3	− 4.6	− 1.1		− 3.7	− 0.4	2.7	0.7
1.30.....	897	− 7.9	+1.7	+ 7.0	+ 2.8	− 2.5	− 0.1	+ 1.2	− 3.2	+ 1.1	+ 1.1	2.9	0.7
1.60.....	526	+ 0.8	+7.6	+ 9.9	+ 6.5	− 2.1	+ 0.4	− 2.1		+ 2.1	−15.	5.2	1.4
2.00.....	245	+ 9.4	+0.4	+ 2.9	0	+ 3.3	+10.	+13.		−11.	−29.	8.8	2.4
2.50.....	43					+ 4.7	+ 9.3	− 7.0	(−65.)	+70.	− 9.3	14.	4.8
3.00.....	(12)									(+50.)	(−100.)	75.	45.

* 1, 2, 3: Flint glass prism: Washington, 1903; Mount Wilson, 1905–1906; Mount Wilson, 1909, June 29, July 7.
4: U. V. glass prism: Mount Wilson, 1909, August 11, 13.
5, 6, 7, 8: Quartz prism: Mount Wilson, 1909, July 8, 9; July 22, 26, 27; August 9, 10; 1910, July 31.
9, 10: Quartz prism: Mount Whitney, 1909, September 3; 1910, August 12, 13, 14.

transmission for other than zero zenith distances, the coefficients here given must be raised to a power equal to the secant of the zenith distance. This does not hold closely for zenith distances above 75°.

TABLE III

Wave-Length	Mean Coefficients of Atmospheric Transmission		
	Washington 1902–1907	Mt. Wilson 1909–1910	Mt. Whitney 1909–1910
0.30	.....	.....	(0.510)
0.325	.....	0.(550)	0.584
0.35	.....	0.612	0.660
0.375	.....	0.662	0.738
0.39	0.445	0.694	0.763
0.42	0.586	0.764	0.806
0.43	0.600	0.778	0.822
0.45	0.640	0.800	0.851
0.47	0.671	0.827	0.880
0.50	0.705	0.858	0.900
0.55	0.739	0.876	0.918
0.60	0.760	0.890	0.934
0.70	0.839	0.942	0.956
0.80	0.865	0.964	0.972
1.00	0.901	0.973	0.980
1.30	0.916	0.972	0.980
1.60	0.930	0.975	0.978
2.00	0.909	0.957	0.940
2.50	0.870	(0.900)	0.930
3.00	.....	.....	0.910

THE EFFECTIVE TEMPERATURE OF THE SUN'S RADIATING LAYER

It has been stated very pertinently[1] that we cannot express solar temperatures either on the thermodynamic scale or on that of the gas thermometer, because no experiments have been made to fix these scales in the range of temperatures prevailing on the sun. All that we know, which is available, is that certain radiation formulae, in which the temperature enters, agree very well with observations on the radiation of the perfect radiator, or so-called "black body," when the gas-thermometer temperatures are introduced. But this verification of these formulae extends only to about 1800° of the absolute Centigrade scale, whereas the solar temperatures seem to be of the order of 6000° or more. We can only compare the solar radiation with that of the perfect radiator,

[1] *Astrophysical Journal*, 19, 35, 1904.

as computed by these formulae, and thus determine the temperature which a "black body" would be assumed to have when its radiation would approximate to that of the sun.

There is another kind of uncertainty in the discussion of the solar temperature. We are not sure that the source of radiation is at the same temperature at the center of the sun's visible disk that it is at the limb, nor that it is the same at a given place on the disk for rays of different wave-lengths. I am of the opinion that the scattering of rays by the molecules of gas in the outer parts of the sun is what limits our depth of view, and that there is no cloudy photosphere. If this is so, we look deeper, and hence to hotter layers, when we view the center of the sun's disk at right angles, than we do when we view the limb; and as molecular scattering is far greater for violet than for red rays, the violet rays which we receive must come from more superficial, and hence cooler, layers, on the whole, than the red ones.[1]

The effect of these two influences on the form of the sun's energy-spectrum-curve may be considerable, and may tend to mislead us greatly as to the probable solar temperature. The first of the two influences considered is the less important, for it leads to no great distortion of the energy-curve, but merely to altering it to correspond approximately to a lower temperature than that which prevails at the deepest layers seen at the center of the visible disk. Much more misleading is the second influence suggested, for it tends to distort the curve so as not to correspond to any "black-body" temperature at all. Considering merely the radiation from the center of the sun's disk, rays of successive wave-lengths (passing from the infra-red toward the ultra-violet) are propagated to us from layers of lower and lower average temperature. Whereas the total radiation of a "black body" varies as the *fourth power* of its temperature, the radiation of the shorter wave-lengths varies as higher powers of the temperature than the fourth, even to the tenth, and still higher powers, as the wave-length decreases beyond that of maximum radiation. Hence, it will be expected: (1) that the solar energy-curve will rise less rapidly from the infra-red side toward the maximum than would

[1] This idea I have elaborated in a book entitled *The Sun*, now in press.

the energy-spectrum-curve of a "black body" of the temperature at which the infra-red rays are prevailingly emitted; (2) that the maximum of solar energy will be shifted toward longer wave-lengths than it would occupy if the distortion did not exist; (3) that in the ultra-violet the solar energy-spectrum will fall off far more rapidly than that of a "black body" generally corresponding in spectral distribution to the solar curve for greater wave-lengths; and (4) that any estimate we may make as to the effective radiating temperature of the sun, based on a simple consideration of the form of its spectral energy-curve, will be too low.

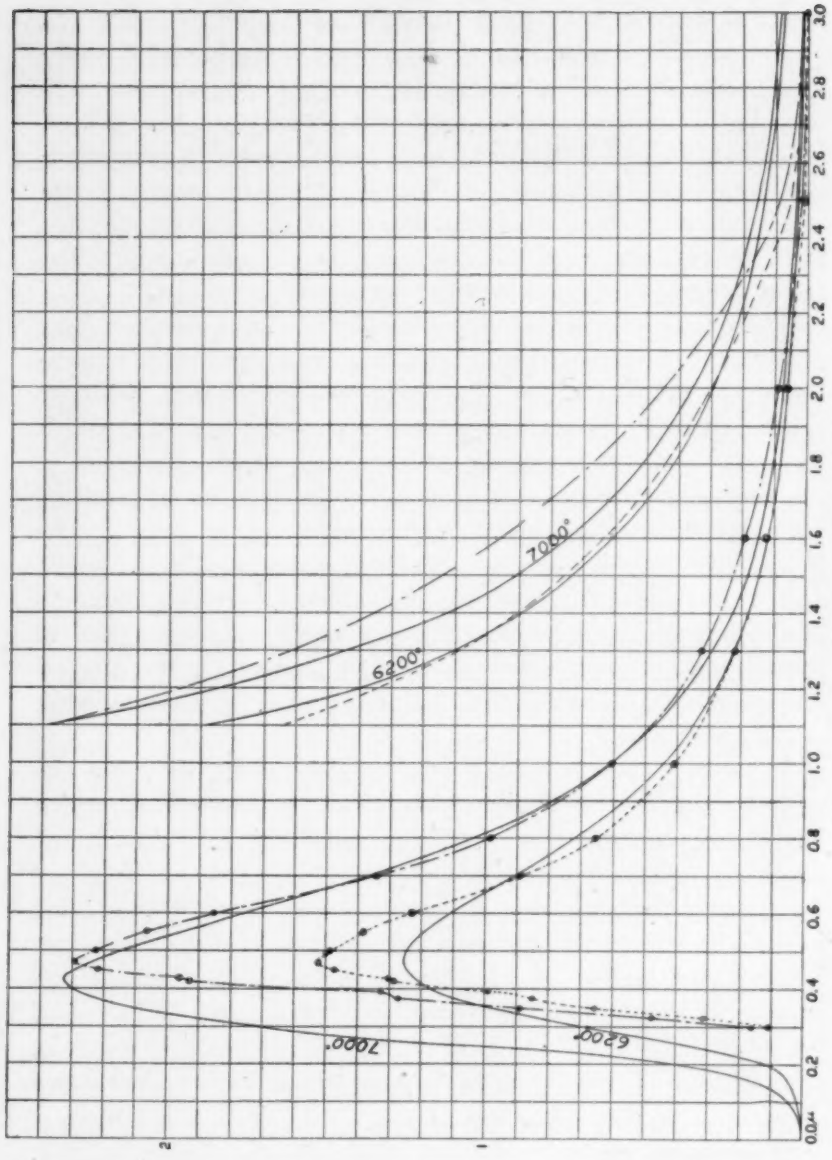
There is another influence tending powerfully to alter the form of the sun's energy-spectrum-curve, namely the selective absorption in the Fraunhofer lines, by virtue of which the rays which are emitted from that average depth below the solar surface at which the continuous spectrum originates, are cut off by absorption in these wave-lengths, and in their places we see only the rays emitted from relatively superficial, and therefore cooler layers. The Fraunhofer lines become so excessively numerous in the violet and ultra-violet solar spectrum that their presence profoundly diminishes the intensity of the sun's energy-spectrum-curve in that spectral region, and thus conspires to magnify the spectral distortion and the underestimation of solar temperatures we have already discussed.

It seems to me, in consideration of these tendencies, that the process to which Goldhammer has given preference,[1] and which is akin essentially to that of Wilsing and Scheiner,[2] is ill-adapted to give a trustworthy estimate of solar temperatures. It gives[3] for the sun temperatures of the order of only 4000°, when applied in the infra-red spectral region, where, as I believe, the temperature is really the highest of all. The diagram shows a comparison between the form of the solar energy-spectrum-curve and those of "black-body" curves at 6200° and 7000° absolute Centigrade, respectively. The solar curve is repeated to match each curve

[1] *Annalen der Physik* (4), **25**, 905–920, 1908.

[2] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Bd. XIX, No. 56, 1909.

[3] *Astrophysical Journal*, **29**, 282, 1909. See also my book *The Sun* above mentioned.


7000°
6200°
7000°
6200°
0.04
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
Energy-spectrum of sun compared with black body

as well as possible in the region $\lambda = 0.5\ \mu$ to $\lambda = 1.2\ \mu$. The longer wave-lengths are repeated on an enlarged scale.

If we take the Wien displacement formula, $\lambda_{max} T = 2930$, and the wave-length of maximum energy as $0.470\ \mu$, the effective solar temperature comes out 6230° absolute Centigrade.

If we take the Stefan formula for total radiation: $E = 76.8 \times 10^{-12}\ T^4$; the solar radius as 696,000 km; the mean radius of the earth's orbit as 149,560,000 km; and the solar constant as 1.922 calories per square centimeter per minute; the effective solar temperature comes out 5830° absolute Centigrade. This last method of computing the solar temperature would perhaps be the best if we could only form some independent estimate of the emissive power of the sun's effective radiating layers. As we are inclined to think that the methods based on the form of the sun's spectrum-energy-curve must necessarily give too low temperature values, and as we find that the method based on total radiation gives still lower ones, it appears that the emissive power of the sun must fall decidedly below that of the perfect radiator.

In view of all this we can only conclude that the sun's effective radiating layer is roughly comparable with a "black body" at 6000° absolute Centigrade, but that the prevailing solar radiating temperatures greatly exceed 6000°, and may even exceed 7000° absolute Centigrade.

SUMMARY

Spectro-bolometric observations extending from the year 1903 to 1910 at Washington, Mount Wilson, and Mount Whitney (altitudes 10, 1750, and 4420 meters, respectively) made with various optical systems, including flint glass, ultra-violet crown glass, and quartz prisms, with mirrors sometimes of silvered glass, sometimes of magnalium, have been used to determine the distribution of energy in the solar spectrum outside the atmosphere. The results do not seem to depend on the altitude of the observing station, but, especially in the ultra-violet, seem to depend somewhat on the character of the spectroscope employed. It is thought that in the ultra-violet spectrum beyond wave-length $0.40\ \mu$ little weight should be given to the results obtained thus far with glass prism spectroscopes.[1]

[1] Better results are expected this summer.

In the ultra-violet region the results here given are obtained with a quartz-magnalium spectroscope.

For the adopted mean normal spectrum-energy-curve the maximum of energy falls at wave-length 0.470 μ.

The intensities at various wave-lengths are as follows:

Wave-Length	μ 0.30	μ 0.35	μ 0.40	μ 0.45	μ 0.47	μ 0.50	μ 0.60	μ 0.80	μ 1.0	μ 1.3	μ 1.6	μ 2.0	μ 2.5	μ 3.0
Intensity....	440	2700	4345	6047	6253	6064	5047	2672	1664	897	526	245	43	12

For additional wave-lengths see the preceding table.

The mean coefficients of atmospheric transmission for vertical rays are given as determined at Washington, Mount Wilson, and Mount Whitney. Coefficients for various wave-lengths are as follows:

Wave-Length		μ 0.35	μ 0.40	μ 0.45	μ 0.50	μ 0.60	μ 0.70	μ 0.80	μ 0.90	μ 1.00	μ 1.50	μ 2.00
Transmission.....	Washington	.....	0.543	0.640	0.705	0.760	0.839	0.865	0.886	0.901	0.922	0.909
	Mt. Wilson	0.612	0.724	0.800	0.858	0.890	0.942	0.964	0.968	0.973	0.974	0.957
Coefficient.....	Mt. Whitney	0.660	0.783	0.815	0.900	0.934	0.956	0.972	0.979	0.980	0.980	0.940

These are smooth-curve values, and do not take account of powerful selective absorption in water-vapor bands. For additional wave-lengths see preceding table.

The form of the spectrum-energy-curve at the earth's surface (not allowing for water-vapor or oxygen bands) may be obtained by multiplying the extra-atmospheric intensities by the corresponding coefficients of atmospheric transmission raised to powers whose exponents are the solar zenith distances.

The temperature of the sun is discussed, and attention is drawn to certain influences which probably tend to distort the sun's spectral energy-curve, and thereby to incline us to set too low values for the probable temperatures prevailing in the sun's radiating layers. The conclusion is drawn from the observations cited that the sun's emission approximates roughly to that of a "black-body" at 6000° absolute Centigrade, but that on account of influences probably active, we ought to assign a solar radiating temperature of the order of 7000° absolute Centigrade.

Astrophysical Observatory
Smithsonian Institution
May 1911

THE ZEEMAN EFFECT FOR VANADIUM[1]

By HAROLD D. BABCOCK

On account of the importance of vanadium in the study of magnetic fields in sun-spots, the following material is presented now. It is preliminary to a fuller discussion of the subject which must be deferred for a period.

The apparatus employed and the manner of taking the photographs have been described in a preceding paper.[2] The field-strengths employed for different plates ranged from 17,500 to 19,800 gausses. For nearly all of the plates lumps of pure vanadium were used as spark terminals, but two photographs were taken with pieces of ferro-vanadium supposed to contain ten per cent of the metal. This substance in the spark yields the vanadium lines in abundance, as well as the stronger iron lines.

In Table I are given the results obtained from all the plates taken up to this time. From λ 5500 to λ 6600 the number of plates upon which the results depend is relatively greater than for the remainder of the spectrum. The wave-lengths employed are those of Hasselberg[3] and of Shaw,[4] except for a very few lines given only by Exner and Haschek.[5] The wave-lengths taken from Shaw's paper are on the Rowland system.

The second column of the table gives the total number of components into which the line appears separated when viewed at right angles to the magnetic field. The number 1, which is occasionally found in this column, indicates that the line is undivided by the field. The letter C, with or without a number, is used here

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 55.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 52; *Astrophysical Journal*, **33**, 217, 1911.

[3] B. Hasselberg, "Die Spectra der Metalle im electrischen Flammenbogen," *Kongl. Svenska Vetenkaps-Akademiens Handlingar*, Bandet **32**, No. 2.

[4] H. Shaw, "The Arc Spectrum of Vanadium," *Astrophysical Journal*, **30**, 127, 1909.

[5] *Wellenlängen Tabellen der Funkenspectren der Elemente*, Leipzig and Vienna, 1902.

as an abbreviation for complex, and signifies that higher resolving power is required for the complete analysis of the components. If a number accompanies the letter it indicates how many groups of components were observed. Measures given for such lines refer to the average separations of the groups of components unless otherwise noted in the remarks.

The third and fourth columns show the separations in Ångströms for the *n*- and *p*-components, respectively, for a field-strength of 20,000 gausses, while the fifth and sixth columns contain for the same field the corresponding values of $\Delta\lambda \cdot 10^8/\lambda^2$.

In some cases it was impossible to measure both members of a pair of components on account of blending with neighboring lines. For such the component clearly shown was referred to the normal position of the line, and the measured difference doubled.

In the "Remarks" column numbers are used to express the relative intensities of the components of a group in case they differ. The relative position of the numbers corresponds to that of the components when viewed with longer wave-lengths on the right. Thus *n* 2:1 means that of the two *n*-components of a line the one on the violet side is twice as intense as the other. No attempt is made to compare the intensities of *n*-components with those of *p*-components. In this column E means that the line is given in Lockyer's list of enhanced lines; −r means not resolved; w with a subscript means widened, the magnitude of the subscript indicating the amount of widening; w_3 represents the greatest widening observed.

As regards types of separation, vanadium is simpler than chromium. It has very few cases of asymmetry and comparatively few lines of more than four components. The largest number of components observed for one line is 12, for λ 6119.74. The one line having ten components is λ 6081.66. There are twenty lines having six components, two each of five, seven, and eight, and eight undivided lines. In addition to these there are 46 quadruple, 349 triple lines, and some whose classification is uncertain.

The average value of $\Delta\lambda \cdot 10^8/\lambda^2$ for all the triplets is 2.20, the extremes being 0.71 and 6.01. For those triplets which lie

TABLE I

SEPARATION OF VANADIUM LINES IN MAGNETIC FIELD

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^5$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
3865.02	3?	0.279	.....	1.87	....	*p* not observed
3867.00	3	0.392	.....	2.62	....	E
3867.77	3	0.357	.....	2.39	....	
3871.23	3	0.326	.....	2.18	....	
3873.80	1?	.....	.....	....	....	
3875.22	3	0.331	.....	2.20	....	
3876.05	6?	0.440	.....	2.93	....	*p*-components uncertain
3876.21	4	0.314	0.173	2.09	1.15	
3878.90	3	0.304	.....	2.02	....	E
3879.82	3	.....	.....	....	....	
3884.04	3	.....	.....	....	....	
3885.00	4	0.240	0.243	1.59	1.61	E
3886.72	3	0.178	.....	1.18	....	
3890.33	4	0.294	0.244	1.94	1.61	
3893.03	3	0.420	.....	2.77	....	
3896.29	1	.....	.....	....	.....	
3897.22	3	0.310	.....	2.04	..	
3898.15	3	0.210	.....	1.38	....	
3899.30	3	0.282	.....	1.85	....	E
3900.33	3	.....	.....	....	.....	Too weak to measure
3901.30	3	0.320	.....	2.10	....	*p*-component not sharp
3902.40	3	0.364	.....	2.39	....	
3903.42	3	0.381	.....	2.50	....	E
3906.89	3	0.412	.....	2.69	....	
3910.01	3	0.442	.....	2.89	....	Double line –r
	3	0.253	.....	1.66		
3910.95	4	0.315	0.116	2.06	0.76	
3912.36	4	0.301	.....	1.97	....	N 1:2:1; 1 *p*-component
		0.000	.....	0.00		
3913.03	3	0.379	.....	2.47	....	
3914.44	3	0.239	.....	1.56	....	E
3916.55	3	0.250	.....	1.63	....	E; *p*-component w_1
3920.15	3	0.408	.....	2.66	.....	*n*-components of this line and the following blended and measured together
3920.65	4	0.408	0.161	2.66	1.05	
3922.05	6	0.484	0.160	3.15	1.04	
		0.186		1.21		
3922.58	3	0.376	.....	2.44	....	
3924.84	3	0.338	.....	2.19	....	
3930.19	3	0.295	.....	1.91	....	
3931.50	3	0.272	.....	1.76	....	
3934.16	3	0.398	.....	2.51	....	Measured normal and red component
3935.28	3	0.185	.....	1.20	....	
3936.42	3	0.579	.....	3.74	....	*p*-component w_3; very weak
3937.68	3	0.429	.....	2.77	....	
3938.35	3	0.292	.....	1.88	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
3939.48	3	0.444	.....	2.86	....	*p*-component w_3 diffuse
3942.16	3	0.446	.....	2.87	....	*p*-component w_3 diffuse
3943.77	3	0.426	.....	2.74	....	
3950.37	3	0.281	.....	1.80	....	
3952.09	3	0.346	.....	2.22	....	E; fringed to red
3963.77	3	0.272	.....	1.73	....	Very difficult
3968.24	1	.....	.....	....	....	
3973.79	3	0.253	.....	1.60	....	E
3979.30	3	0.345	.....	2.18	....	
3979.59	3	0.272	.....	1.72	....	
3980.66	3	0.263	.....	1.66	....	
3984.75	4	.....	0.240	....	1.52	*n*-components too weak to measure
3985.90	3	0.296	.....	1.86	....	E
3988.97	4	.....	0.210	....	1.32	*n*-components too weak to measure
3990.71	3?	0.265	.....	1.66	....	
3992.95	3	0.307	.....	1.93	....	
3997.30	3	0.298	.....	1.86	....	E
3998.87	3	0.321	.....	2.01	....	
3999.30	3	0.296	.....	1.85	....	E
4003.10	6	0.438 0.074	0.367	2.73 0.46	2.29	
4005.86	3	0.361	.....	2.25	....	E
4017.00	3	0.276	.....	1.71	....	E
4023.50	3	0.310	.....	1.92	....	E
4031.37	4	0.395	.....	2.43	....	2 *p*-components too weak to measure
4031.98	3	0.330	.....	2.03	....	
4035.77	3	0.227	.....	1.39	....	E
4036.93	4	0.373	0.242	2.29	1.48	*n*-components; w_3 not sharp
4042.78	3	0.288	.....	1.76	....	*p*-component w_2
4048.77	3	0.341	.....	2.08	....	Red component wider than violet
4051.11	3	0.383	.....	2.33	....	Measured normal and violet component
4051.48	3	0.391	.....	2.38	....	Measured normal and red component
4053.80	3	0.302	.....	1.84	....	E; not sharp
4057.21	3	0.303	.....	1.84	....	
4064.09	3	0.202	.....	1.22	....	
4065.20	3	0.282	.....	1.71	.....	E; *p*-component w_1
4067.90	3	0.366	.....	2.21	....	Difficult; *p*-component w_2
4071.67	3	0.318	.....	1.92	....	*p*-component w_1
4090.70	3	0.352	.....	2.10	....	
4092.09	4	0.270	0.283	1.61	1.69	*n* 1:2
4092.54	3	0.460	.....	2.75	....	Components shifted 0.048 Å to red; w_3
4092.83	3	0.445	.....	2.66	....	
4093.65	4	0.429	0.414	2.56	2.49	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^5$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4094.42	3	0.374	.....	2.23	....	Difficult; components w_3
4095.64	3	0.331	.....	1.97	....	
4097.09	4?	.....	0.191	....	1.14	*n*-components not measurable
4098.54	4?	.....	0.396	....	2.36	*n*-components not measurable
4099.93	3	0.454	.....	2.70	....	
4102.32	3	0.275	.....	1.63	....	
4104.55	3	0.419	.....	2.49	....	
4104.92	3	0.435	.....	2.58	....	Measured normal and violet component
4105.32	3	0.466	.....	2.77	....	
4107.64	4?	.....	0.442	....	2.62	*n*-components not measurable
4108.36	3	0.236	.....	1.40	....	
4109.94	6	0.791 0.340	0.227	4.68 2.01	1.34	
4111.92	3	0.467	.....	2.76	....	
4112.47	3	0.344	.....	2.04	....	Measured normal and red component
4113.65	3	0.362	.....	2.14	....	w_1
4115.32	3	0.490	.....	2.89	....	
4116.64	5	1.026 0.511	.....	6.20 3.02	....	See description in text
4118.34	3	0.362	.....	2.14	....	w_1; measured normal and violet component
4118.73	4	0.425	0.182	2.51	1.07	Diffuse; measured normal and red component
4119.58	4	0.396	0.242	2.33	1.43	Components w_2; not sharp
4120.69	6	0.365 0.149	0.334	2.15 0.88	1.97	Weak; not sharp
4123.65	6	0.810 0.345	0.245	4.76 2.03	1.44	Outer *n*-components lightly fringed outside
4124.23	4	0.354	0.182	2.08	1.07	
4128.25	3	0.475	.....	2.79	.....	
4129.00	3	0.149	. ...	0.87	....	Not sharp
4132.13	3	0.470	.....	2.75		
4133.92	3	0.483	.....	2.83	....	
4134.61	3	0.471	.....	2.75	....	
4136.25	4	0.346	0.165	2.02	0.96	
4139.39	3	0.468	.....	2.73	....	
4149.02	3	0.358	.....	2.08	....	
4150.84	3	0.334	.....	1.94	....	
4159.84	1	.....	.....	....	....	w_3; not sharp
4171.45	4	0.419	0.152	2.41	0.87	
4174.18	3	0.384	.....	2.20	....	
4177.25	3	0.460	.....	2.64	....	Very difficult; w_2
4178.50	3	0.306	...	1.75	....	E; w_2; not sharp
4179.53	4	.....	0.529	....	3.03	*n*-components not measurable; *p*-components fringed inside

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^3$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4182.23	3	0.310	.....	1.77	....	
4182.74	1	.....	.....	....	....	w_3; not sharp
4183.59	3	0.360	.....	2.06	....	E
4191.70	4	0.516 0.000	.....	2.94 0.00	....	*n* 3:2:3; 1 *p*-component
4197.77	3	0.340	.....	1.93	....	
4202.52	3	0.395	.....	2.24	....	E; w_1
4204.67	4	0.524	.....	2.96	....	Weak, not sharp, w_2
4205.23	3	0.332	.....	1.88	....	E
4209.98	4	0.480	.....	2.71	....	*p*-components not measurable; w_3; not sharp
4224.30	3	0.501	.....	2.81	....	
4225.40	3	0.263	.....	1.47	....	E
4226.78	3	0.540	.....	3.02	....	Weak; measured normal and violet component
4227.90	3	0.334	.....	1.87	....	
4232.20	3	0.451	.....	2.52	....	E
4232.62	3	0.451	.....	2.52	....	
4233.09	3	0.415	.....	2.32	....	
4234.12	3	0.360	.....	2.01	....	
4234.70	4	0.328	0.185	1.83	1.03	Not sharp
4235.90	3	0.128	.....	0.71	....	
4243.10	3	0.326	.....	1.81	....	E; not sharp
4247.46	3	0.468	.....	2.60	....	*p*-component w_2
4254.55	3	0.435	.....	2.40	....	
4257.53	3	0.516	.....	2.85	....	
4259.46	3	0.416	.....	2.29	....	
4262.32	3	0.533	.....	2.94	....	
4265.28	3	0.395	.....	2.17	....	
4267.50	3	0.526	.....	2.89	....	
4268.78	3	0.414	.....	2.27	....	
4269.92	3?	0.670	.....	3.68	....	*p*-component w_3
4270.49	3	0.447	.....	2.45	....	
4271.71	3	0.384	.....	2.10	....	
4277.12	3	0.330	.....	1.80	....	
4284.19	3	0.238	.....	1.30	....	
4286.57	3	0.520	.....	2.83	....	Difficult
4287.97	3	0.385	.....	2.10	....	
4289.89	3	0.572	.....	3.11	....	*p*-component diffuse
4291.97	3	0.381	.....	2.07	....	
4296.28	3	0.354	.....	1.92	....	
4297.86	3	0.329	.....	1.78	....	
4298.17	3	0.298	.....	1.61	....	*p*-component w_2
4302.30	3	0.329	.....	1.78	....	
4305.63	3	0.411	.....	2.22	....	
4306.35	3	0.523	.....	2.82	....	
4307.33	3	0.521	.....	2.81	....	w_2; diffuse
4309.95	3	0.517	.....	2.78	....	
4330.18	3	0.143	.....	0.76	....	
4332.98	3	0.354	.....	1.88	....	
4341.15	3	0.430	.....	2.28	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2} \cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4343.00	3	0.325	.....	1.72	....	
4346.59	3	0.305	.....	1.61	....	*p*-component diffuse; difficult
4353.02	3	0.469	.....	2.48	....	
4355.09	3	0.393	.....	2.07	....	
4356.10	3	0.613	.....	3.23	....	*p*-component w_3
4363.69	3	0.560	.....	2.94	....	
4364.37	3	0.191	.....	1.00	....	
4368.25	3	0.560	.....	2.94	....	*p*-component w_1
4368.76	3	0.357	.....	1.87	....	
4373.40	3	0.398	.....	2.08	....	
4373.99	3	0.396	.....	2.07	....	
4375.47	3	0.386	.....	2.02	....	*n*-component w_2
4379.38	3	0.449	.....	2.34	....	Very strong, w_3
4380.69	3	0.457	.....	2.38	....	Measured normal and violet component
4384.87	3	0.429	.....	2.23	....	
4390.13	3	0.405	.....	2.10	....	Very strong, w_3
4392.24	3	0.636	.....	3.30	.....	
4394.01	3	0.348	.....	1.80	....	w_2
4395.40	3C	0.396	.....	2.05	....	
4399.63	3	0.310	.....	1.60	....	Difficult; *p*-component w_2
4400.74	5	0.785 0.000	0.409	4.05 0.00	2.11	*n* 1:3:1
4403.86	3	0.453	.....	2.34	....	
4406.30	3	0.395	.....	2.04	....	*p*-component diffuse
4406.80	4?	0.537	.....	2.76	....	2 strong *p*-components –r w_2; not well resolved
4407.85	4	0.573	0.228	3.09	1.17	*n*-components blended with near-by lines; very strong, w_2
4408.36	4	.....	0.329	....	1.69	
4408.67	6	0.676 0.309	0.471	3.48 1.59	2.42	*n* 1:2:2:1; *n*-components shifted 0.09 Å to red; w_2
4412.30	3	1.050	.....	5.40	....	*p*-components w_3; not sharp
4416.63	6	1.110 0.223	0.439	5.69 1.14	2.25	*n* 2:1:1:2
4420.08	3	0.639	.....	3.27	....	
4421.73	3C	0.894	.....	4.57	....	
4423.32	3	0.408	.....	2.09	....	Probably double line; components blended; difficult
4423.41	3?	.....	.....	....	....	See preceding line
4424.74	3	0.374	.....	1.91	....	Very weak
4425.86	1?	.....	.....	....	....	
4426.17	3	0.729	.....	3.72	....	*p*-component w_2
4428.68	3	0.677	.....	3.45	....	
4429.95	3	0.655	.....	3.34	....	*p*-component w_3; not sharp

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^3$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4430.68	3	0.900	.....	4.58	....	
4434.80	3	0.425	.....	2.16	....	Diffuse
4436.31	6	1.029 0.682	0.189	5.23 3.47	0.96	*n* 1:3:3:1
4438.02	3?	0.594	.....	3.02	....	*p*-component w_3
4441.88	4	0.641	0.206	3.25	1.04	w_1
4443.52	3	0.367	.....	1.86	....	Measured normal and red component
4444.40	4	0.768	0.282	3.89	1.43	w_1
4449.77	3	0.355	.....	1.79	....	
4451.09	3	0.367	.....	1.85	....	
4452.19	3	0.399	.....	2.01	....	
4457.65	8?	0.440 0.197	.....	2.22 0.99	....	*p*-component w_3, sharp, probably four parts –r
4457.97	3	0.365	.....	1.84	....	Measured normal and red component
4459.93	3	0.374	.....	1.88		*p*-component w_2
4460.46	3	0.459	.....	2.31	....	w_2
4462.56	3	0.373	.....	1.87	....	
4468.19	3	0.524	.....	2.63	....	w_2, diffuse
4469.88	3	0.369	.....	1.85	....	
4474.21	3	0.427	.....	2.13	....	
4474.89	3	0.391	.....	1.95	....	
4480.20	4	0.405	0.240	2.02	1.20	Weak and diffuse
4489.06	3	0.375	.....	1.86	....	w_2
4490.95	3	0.476	.....	2.36	....	
4496.26	3	0.393	.....	1.94	....	
4502.12	3	0.425	.....	2.10	....	
4509.49	4?	.....	0.450	....	2.21	*n*-components not measurable
4513.79	3	0.470	.....	2.31	.....	
4514.36	3	0.336	.....	1.65	....	
4520.67	4?	.....	0.247	....	1.21	*n*-components not measurable
4524.38	3	0.440	.....	2.15	....	*p*-component w_2; slightly shifted to violet
4525.31	5	0.785 0.377	.....	3.84 1.84	....	*p*-component w_3, diffuse. Also *Fe* line
4528.16	3	0.458	.....	2.23	....	Measured normal and violet component
4528.66	3	0.460	.....	2.24	....	E
4529.47	3	0.392	.....	1.91	....	Difficult
4529.76	3	0.288	.....	1.40	....	
4530.97	3	0.436	.....	2.12	....	
4537.84	3	0.458	.....	2.22	....	
4540.18	3?	0.595	.....	2.89	....	w_2, diffuse
4545.57	3	0.425	.....	2.06	....	
4549.81	3	0.395	.....	1.91	....	
4552.05	3	0.409	.....	1.97	....	
4553.25	3	0.385	.....	1.86	....	
4560.90	3	0.399	.....	1.92	.....	

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^5$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4564.76	3	0.378	.....	1.81	....	E, w_1
4570.60	4	0.465	0.252	2.23	1.21	
4571.96	3	0.366	.....	1.75	....	
4577.36	3	0.282	.....	1.35	....	
4578.92	3	0.307	.....	1.46	....	
4579.38	4	0.478	0.286	2.28	1.36	
4580.57	3	0.360	.....	1.72	....	
4583.96	4	0.461	0.383	2.19	1.82	Diffuse
4586.54	3	0.412	.....	1.96	....	
4590.70	3	0.470	.....	2.23	....	
4591.39	3	0.406	.....	1.93	....	
4594.27	3	0.442	.....	2.09	....	
4600.34	3	0.317	.....	1.50	....	E, diffuse
4606.33	4?	.....	0.425	....	2.00	*n*-components not measurable
4609.84	4	0.463	0.220	2.18	1.04	
4611.10	4?	.....	0.362	....	1.70	*n*-components not measurable
4619.85	3	0.392	.....	1.84	....	*n*-components of this line and the following blended and measured together
4619.97	4	0.392	0.267	1.84	1.25	
4624.62	4	0.583	0.346	2.73	1.62	w_2
4626.67	4	0.545	0.535	2.55	2.50	
4635.35	4	0.508	0.247	2.37	1.15	Diffuse
4640.25	4	0.144	0.311	0.67	1.44	
4640.92	4	0.635	0.181	2.95	0.84	
4644.64	3	0.540	.....	2.50	....	
4646.59	3	0.408	.....	1.89	....	
4670.66	3	0.487	.....	2.23	....	
4687.10	3	0.616	.....	2.80	....	*p*-component w_2
4699.52	3	0.463	.....	2.10	....	
4705.26	3	0.487	.....	2.20	....	
4706.34	3	0.553	.....	2.50	....	
4706.75	3	0.443	.....	2.00	....	
4707.62	3	0.443	.....	2.00	....	
4710.74	3	0.436	.....	1.96	....	
4714.28	3	0.400	.....	1.80	....	
4716.08	3	0.391	.....	1.76	....	
4717.85	3	0.345	.....	1.55	....	
4721.70	3	0.431	.....	1.93	....	
4723.06	3	0.494	.....	2.22	....	
4729.73	3	0.764	.....	3.41	....	
4730.57	3	0.436	.....	1.95	....	
4742.79	3	0.389	.....	1.73	....	
4746.81	4?	.....	0.684	....	3.04	*n*-components very diffuse
4748.70	4?	.....	0.427	....	1.89	*n*-components very diffuse
4751.16	4	0.549	0.361	2.43	1.60	Diffuse

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^3$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4751.75	3	0.683	.....	3.02	....	Diffuse; measured normal and red component
4754.13	4	0.578	0.258	2.56	1.14	Diffuse
4757.68	1	.....	.....	....	....	
4766.80	3	0.296	.....	1.30	....	
4776.54	3	0.380	.....	1.67	....	Violet component blended with that of the following
4776.70	3	0.532	.....	2.33	....	
4786.70	3	0.454	.....	1.98	....	
4797.07	3	0.490	.....	2.13	....	
4807.70	3	0.512	.....	2.22	....	
4827.62	4	0.598	0.228	2.57	0.99	
4831.80	4	0.520	0.350	2.23	1.50	Components w_2
4832.59	7	.....	0.541	....	2.32	Probably 5 *n*-components −r
4851.65	3	0.224	.....	0.95	....	
4864.93	3	0.405	.....	1.71	....	
4875.66	3	0.486	.....	2.04	....	
4881.75	3	0.526	.....	2.21	....	
4900.84	3	0.486	.....	2.02	....	
4904.59	3	0.500	.....	2.08	....	
4925.83	3	0.741	.....	3.05	....	
4932.24	3	0.725	.....	2.98	....	
5002.54	3	0.450	.....	1.80	....	
5014.83	3	0.436	.....	1.74	....	
5064.32	3	0.489	.....	1.91	....	
5105.37	3	0.431	.....	1.65	....	
5128.71	3	0.597	.....	2.27	....	
5138.58	3	0.566	.....	2.14	....	
5139.74	3	0.427	.....	1.62	....	
5148.95	3	0.532	.....	2.01	....	
5159.56	3	0.393	.....	1.48	....	w_2; diffuse
5170.15	3	.....	0.512	....	1.92	1 *n*-component, 2 *p*-components
5177.03	3	0.530	.....	1.98	....	
5192.22	3	0.317	.....	1.18	....	
5193.18	3	0.735	.....	2.73	....	
5193.82	3	0.495	.....	1.84	....	Measured normal and red component
5195.01	3	0.664	.....	2.46	....	
5195.58	3	0.612	.....	2.27	....	
5216.80	3	0.780	.....	2.87	....	
5225.97	3	0.781	.....	2.86	....	
5233.91	3	0.763	.....	2.78	....	
5234.31	3	0.458	.....	1.67	....	
5241.06	3	0.552	.....	2.01	....	
5385.39	3	0.633	.....	2.18	....	
5402.17	3	0.524	.....	1.80	....	
5415.51	3	0.579	.....	1.98	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
5434.43	3	0.631	.....	2.14	....	
5487.48	3	0.712	.....	2.36	....	
5488.18	3	0.650	.....	2.16	....	
5490.22	3	0.576	.....	1.91	....	
5505.13	6	0.942 0.372	0.324	3.11 1.23	1.07	*n* 1:3:3:1; poor
5507.97	3	0.452	.....	1.49	....	
5511.41	3	0.942	.....	3.10	....	
5546.18	3	1.083	.....	3.52	....	Measured normal and violet component
5547.31	3	1.032	.....	3.35	....	
5559.00	6	0.977 0.368	0.258	3.16 1.19	0.83	*n* 1:3:3:1
5561.92	4	0.864	.....	2.79	....	*p*-components too weak to measure
5584.75	3	0.898	.....	2.88	....	
5585.00	3	0.696	....	2.23	....	
5586.26	3	0.642	.....	2.06	....	
5588.71	3	0.778	.....	2.49	....	
5592.67	3	0.901	.....	2.88	....	
5601.63	3	0.648	.....	2.06	....	
5605.20	6	1.086 0.347	0.349	3.46 1.10	1.11	*n* 1:3:3:1
5624.80	3	0.824	.....	2.60	....	
5625.16	3	0.692	.....	2.19	....	
5626.27	1	.....	.....	....	....	
5627.86	3	0.863	.....	2.72	....	
5635.76	3	0.532	.....	1.67	....	
5646.36	6	1.080 0.350	0.363	3.39 1.10	1.14	*n* 3:1:1:3
5657.11	3?	0.453	.....	1.42	....	Measured normal and violet component; doubtful
5657.67	3	0.926	.....	2.89	....	
5668.61	3	0.916	.....	2.85	....	
5671.10	3C	0.400	.....	1.24	....	Strong, w_3; fringed
5683.47	4	0.696	.....	2.16	....	*p*-components too weak to measure
5698.74	3	0.593	.....	1.83	....	Strong, w_2
5703.83	3	0.571	.....	1.75	....	
5707.26	3	0.347	.....	1.06	....	Strong, w_2
5709.25	3	0.582	.....	1.78	....	
5716.49	3	0.706	.....	2.16	....	
5725.90	3	0.656	.....	2.00	....	
5727.25	3	0.700	.....	2.13	....	
5727.90	7	0.973 0.523 0.000	0.687	2.97 1.60 0.00	2.09	
5733.34	3	0.574	.....	1.75	....	
5734.26	3	0.598	.....	1.82	....	
5737.28	4	0.766	0.478	2.33	1.45	w_3

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^5$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
5743.67	4	0.820	0.496	2.48	1.50	w_2
5747.98	3	0.684	.....	2.07	....	
5749.13	3	0.556	.....	1.68	....	
5750.90	3	0.839	.....	2.54	....	
5752.99	6	1.229 0.320	0.412	3.71 0.97	1.24	*n* 1:3:3:1
5761.70	6	1.810	0.820 0.264	5.45	2.47 0.80	
5772.66	3	0.724	.....	2.17	....	*n*-components lightly fringed outside
5776.95	4	0.576	0.779	1.72	2.33	
5783.76	3	0.477	.....	1.42	....	
5784.64	3	0.718	.....	2.14	....	
5786.42	3	0.630	.....	1.88	....	
5788.85	4	0.896	0.842	2.67	2.51	
5807.40	3	0.541	.....	1.60	....	w_2
5817.33	4	0.487	0.500	1.44	1.48	
5830.97	3	0.732	.....	2.15	....	Strong, w_2, not sharp
5846.56	3	0.796	.....	2.33	....	Not sharp, w_2
5850.60	4	0.976	0.356	2.85	1.04	
5924.80	3	0.775	.....	2.21	....	
5979.18	3	0.688	.....	1.92	....	
5981.03	3	0.682	.....	1.91	....	w_2
6002.60	3	0.782	.....	2.17	....	
6002.83	3	1.370	.....	3.80	....	
6008.97	4	0.839	0.955	2.32	2.65	
6016.38	3	0.698	.....	1.93	....	
6018.16	4?	.....	0.506	....	1.40	*n*-components not measurable
6022.02	3	0.708	.....	1.95	....	
6025.66	3	0.626	.....	1.72	....	
6039.95	4	0.992	0.341	2.72	0.93	
6054.78	3	0.615	.....	1.68	....	
6058.36	6	1.762 0.585	0.577	4.80 1.59	1.57	*n* 1:3:3:1
6063.56	3	0.521	.....	1.42	....	
6067.48	3	0.646	.....	1.76	....	
6081.66	10	1.372 1.007 0.621	0.552	3.71 2.72 1.68	1.49	*n* 2:3:2:2:3:2; inner *p*-components too weak to measure
6090.43	3C	0.857	.....	2.31	....	
6107.20	3	0.707	.....	1.90	....	
6111.85	4	0.878	0.895	2.35	2.40	
6119.74	12	0.719	0.388 0.135	1.92	1.04 0.36	*n* 1:2:3:4:4:3:2:1
6128.56	5	1.498 0.000	0.617	3.99 0.00	1.64	
6135.56	6	1.325 0.359	0.500	3.52 0.95	1.33	*n* 1:3:3:1

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^6$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
6150.35	3	0.846	.....	2.24	....	
6170.54	3	0.808	.....	2.12	....	
6189.57	3	0.719	.....	1.88	....	W_2
6190.67	4?	.....	0.840	....	2.19	*n*-components not measurable; *p*-components fringed inside
6199.38	3	1.054	.....	2.74	....	
6214.06	4	1.078	0.329	2.79	0.85	
6216.55	3	1.057	.....	2.74	....	
6218.55	3	0.959	.....	2.48	....	
6221.54	4?	.....	0.390	....	1.01	*n*-components uncertain
6224.71	4	1.072	0.393	2.77	1.01	
6230.98	3	1.018	.....	2.62	....	*n*-components fringed outside
6233.39	4	1.048	0.502	2.70	1.29	
6236.52	3	0.744	.....	1.91	....	
6240.35	8	1.545 1.092 0.618	0.762	3.97 2.80 1.59	1.96	
6243.02	4	1.314	0.557	3.37	1.43	
6243.29	3	1.151	.....	2.95	....	
6245.43	4	1.076	1.421	2.76	3.64	
6252.02	3	1.133	.....	2.90	....	
6257.12	3	1.188	.....	3.03	....	
6258.79	3	2.354	.....	6.01	....	
6261.42	6	2.310 0.571	0.820	5.89 1.46	2.09	*n* 3:2:2:3
6266.54	3C	1.688	.....	4.30	....	
6269.05	3	1.406	.....	3.58	....	
6274.85	6	1.821 0.925	0.457	4.62 2.35	1.16	*n* 1:3:3:1
6282.57	3	0.858	.....	2.17	....	
6285.38	3	1.156	.....	2.93	....	
6293.03	3	1.114	.....	2.81	....	
6296.70	3	1.131	.....	2.85	....	
6309.98	3	1.004	.....	2.52	....	
6311.74	3	0.790	.....	1.98	....	
6321.47	3	1.152	.....	2.88	....	W_2
6324.93	3	0.828	.....	2.07	....	W_2
6327.05	3	0.834	.....	2.08	....	
6339.32	3	0.781	.....	1.94	....	
6349.68	3	0.701	.....	1.74	....	
6355.83	3	0.673	.....	1.66	....	
6357.53	3	0.589	.....	1.45	....	
6359.05	3	0.833	.....	2.06	....	
6361.51	3	0.788	.....	1.95	....	
6374.74	3	0.487	.....	1.20	....	
6379.58	3	0.734	.....	1.80	....	
6393.56	3	0.598	.....	1.46	....	
6430.75	3	0.990	.....	2.39	....	
6431.90	3	0.958	.....	2.32	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
6433.44	3	0.808	.....	1.95	....	
6438.35	3	0.863	.....	2.08	....	
6452.56	3	1.153	.....	2.77	....	
6488.33	3	0.877	.....	2.08	....	
6504.40	6	1.746 1.040	0.418	4.14 2.46	0.99	*n* 1:3:3:1
6531.66	3	1.262	.....	2.96	....	
6550.33	3	0.876	.....	2.04	....	
6558.28	3	0.920	.....	2.14	....	
6566.13	3	2.125	.....	4.93	....	
6606.20	6	..... 1.104	0.398	.... 2.53	0.91	Outer *n*-components too weak to measure
6625.05	3	1.125	.....	2.56	....	

between λ 3850 and λ 4800 the average is 2.17, which is the same as the corresponding quantity for chromium.[1] As in the case of the latter metal, so for vanadium, the values of $\Delta\lambda \cdot 10^8/\lambda^2$ for the triple lines occur in two main groups, of which one is much larger than the other. The first column of Table II shows a series of intervals in the values of $\Delta\lambda \cdot 10^8/\lambda^2$, while the second and third columns contain the numbers of values occurring in the intervals. The enhanced lines are given here separately in order to show that their grouping is not peculiar.

The average of all the values lying between 1.40 and 2.50 is 1.99 ± 0.011, while the average for the second group, between the limits 2.50 and 3.20, is 2.78 ± 0.012, the appended quantities being probable errors. For the enhanced lines the average is 1.95. Although the mean for the enhanced lines should be compared with the mean of those not enhanced, rather than with the general average, it does not appear that the enhanced lines are distinctly different from the others in their magnetic separation. The same conclusion was reached in the case of chromium.

For 46 quadruple lines the average value of $\Delta\lambda \cdot 10^8/\lambda^2$ for the *n*-components is 2.38, for the *p*-components 1.45.

There are a few peculiar lines in Table I to which attention should be called. The first is λ 4116.64, which has four *n*-com-

[1] *Op. cit.*

ponents and only one *p*-component. The outer *n*-components show a very large separation and are shifted (by a field of 20,000 gausses) 0.033 Å to the red, while the inner pair are shifted 0.041 Å to the violet, the mean position of all four coinciding with the normal. $\lambda\lambda$ 3912.36 and 4191.70 each have three *n*-components and only one *p*-component. λ 5170.15 shows only one *n*-component, but two *p*-components—if other components exist they are very weak. λ 5761.70 is an interesting sextuple line having two widely separated *n*-components and four *p*-components. λ 5776.95 is a quadruple line of which the *p*-components are more widely separated than the *n*-components, in the ratio 4:3, while λ 4640.25 of the same type has its *p*-components twice as far apart as its *n*-components.

TABLE II

DISTRIBUTION OF TRIPLET SEPARATIONS

INTERVALS IN $\frac{\Delta\lambda}{\lambda^2} \cdot 10^3$	NUMBER OF VALUES	
	Not Enhanced	Enhanced
0.60–0.80	2	..
0.80–1.00	2	..
1.00–1.20	4	..
1.20–1.40	7	1
1.40–1.60	11	3
1.60–1.80	37	5
1.80–2.00	59	9
2.00–2.20	67	2
2.20–2.40	38	4
2.40–2.60	17	2
2.60–2.80	27	1
2.80–3.00	28	..
3.00–3.20	6	..
3.20–3.40	5	..
3.40–3.60	4	..
3.60–3.80	2	..
3.80–6.20	5	..

For lines having more than three components various ratios of separation occur. These are collected in Table III. Ciphers in this table indicate central components coinciding with the normal position of the line.

The order of the ratios in any column corresponds to the frequency with which they occur, the most common ratio being at the top.

TABLE III

RATIOS OF SEPARATION

Total Number of Components						
4	5	6	7	8	10	12
2:1	0:2:5	1:1:3	0:3:4:6	4:5:7:10	?:1:1:2:3	1:3:6:?:?:?
3:1	0:1:2	2:3:8				
1:1		2:3:7				
5:3		1:3:7				
3:2		1:2:2				
3:4		1:2:4				
1:2		1:2:5				
		1:4:6				
		1:5:6				
		2:3:4				
		2:5:8				

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MOUNT WILSON SOLAR OBSERVATORY
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THE INFLUENCE OF A MAGNETIC FIELD UPON THE SPARK SPECTRA OF IRON AND TITANIUM—SUMMARY OF RESULTS[1]

By ARTHUR S. KING

The purpose of this paper is to give an outline of a detailed study of the Zeeman effect for iron and titanium carried out in the Pasadena Laboratory of the Solar Observatory, which is being published in full under the above title by the Carnegie Institution of Washington in Vol. II of the *Papers of the Mount Wilson Solar Observatory*.

The range of wave-length covered is from λ 3660 to about λ 6700, slightly beyond this for titanium. Between these limits, 662 lines in the spark spectrum of iron and 458 in that of titanium have been treated, at least to the extent of describing their type of separation by the magnetic field.

The spectra of both iron and titanium have been investigated previously to some extent. Measurements for portions of the iron spectrum have been published by H. M. Reese,[2] N. A. Kent,[3] W. Hartmann,[4] and Mme. H. B. van Bilderbeek-van Meurs.[5] These measurements are confined for the most part to the blue and violet regions and cover only the stronger lines. For titanium, extensive lists have been published by J. E. Purvis[6] and by the author.[7] The former list is mainly for the ultra-violet, including the stronger

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 56.

[2] "An Investigation of the Zeeman Effect," *Astrophysical Journal*, **12**, 120, 1900.

[3] "Notes on the Zeeman Effect," *Astrophysical Journal*, **13**, 288, 1901.

[4] "Das Zeeman Phaenomen im sichtbaren Spectrum von Kupfer, Eisen, Gold, und Chrom," Dissertation, Halle, 1907.

[5] "Magnetische Splitsing van het Ultraviolette IJzerspectrum," Dissertation Amsterdam, 1909.

[6] "The Influence of a Strong Magnetic Field on the Spark Spectra of Titanium, Chromium, and Manganese," *Proceedings Cambridge Philosophical Society*, **14** (1), 41, 1906.

[7] "The Zeeman Effect for Titanium," *Contributions from the Mount Wilson Solar Observatory*, No. 39; *Astrophysical Journal*, **30**, 1, 1909.

lines. The present investigation includes about 50 per cent more lines than the previous one by me, the data being compiled from a much more extensive set of plates, taken with higher dispersion and stronger field. The gain in all points is so great that these measures may be taken as superseding the former list.

The weaker lines in the two spectra have been photographed as far as they were obtainable, an object which has added to the labor and altered to some extent the experimental methods which would be used for the stronger lines alone. The tables for titanium contain all but the weakest of those lines given in the regular lists of arc and spark lines. As much cannot be claimed for iron, however, as numerous lines, fairly strong in the arc, are not brought out by the spark in the magnetic field even with an exposure of many hours. This is especially true of those lines of diffuse appearance, which are particularly numerous in the iron spectrum.

The publication summarizes the theory and the results of former investigations on the Zeeman effect so far as they apply directly to the present research. The apparatus and experimental methods are described in detail. The spark was given by a 5 K.W. transformer, the terminals being held between the poles of a Du Bois electro-magnet. The light was usually taken at right angles to the lines of magnetic force and passed through a Nicol prism placed above the slit of the vertical Littrow spectrograph. By rotating the Nicol through 90°, the components given by light vibrating parallel to the lines of force and in a plane perpendicular to the force-lines were successively photographed. A variety of field-strengths were used, sometimes slightly over 20,000 gausses, but all values for the tables were reduced to standard fields of 16,000 gausses for the iron spectrum and 17,500 gausses for that of titanium. A system of checking the field-strengths for different plates, somewhat difficult in the case of iron, was carried out, so that considerable confidence can be placed in the uniformity of the field for all parts of the spectrum.

The Littrow spectrograph was used with objectives of either 13 ft. (4 m) or 30 ft. (9.1 m) focal length, the third order of the plane grating generally being used at the shorter focus and the second order at the longer. Scales of about 1.3 and 0.95 Ångström

units per mm, respectively, were obtained for photographs with the two arrangements, but dispersions as high as 0.62 and 0.44 Å per mm were occasionally employed with higher orders of the grating.

The photographic processes were adapted as nearly as possible to the somewhat conflicting requirements of speed, moderately fine grain, contrast sufficient to give sharply defined lines and still not such as to give too large differences in intensity between the stronger and weaker lines. For the most part, the Seed "27" plate was used for the blue end of the spectrum, and for wave-lengths greater than 4600 the same plate was sensitized by a three-dye bath. Regions of the spectrum containing a wide range in the intensity of lines often required special treatment both as to instrumental and photographic methods.

In the measurement of the plates, I am indebted to Miss Wickham and to Miss Griffin for much assistance.

In making extracts from the tables, there is no criterion which makes a certain class or type of lines of special importance for a summary of this sort, so the method adopted is to take the descriptions of lines for various short stretches of wave-length bodily from the complete tables. These regions are selected to represent as well as possible the types of separation occurring throughout the two spectra. Each region contains a number of strong lines and includes some complex types of separation. The change in average separation with the wave-length may be noted by comparing the general magnitude of the measurements in successive regions.

EXPLANATION OF THE TABLES

The letters *n* and *p* in the tables are used to denote the Zeeman components given by light vibrations in a plane at right angles to the lines of magnetic force and parallel to these respectively. The wave-lengths are on the Rowland system. The second column gives a rough value for the intensity of lines in the spark spectrum, taken partly from the tables of Exner and Haschek and partly estimated on the same scale from my plates. Weak lines are graded 1, but there is a considerable variation in the strength of lines which are given this value.

The third column gives the best judgment of the type of separation that can be made from the photographs. The description given must be considered in connection with the measured separation and widening of components given in the columns for $\Delta\lambda$ of the *n*- and *p*-components, and is usually made clear by these. Frequently a supplementary remark is needed in the case of complex lines. The interrogation point is freely used to indicate that the line is probably of the character given, though not clearly shown to be so on the plates. The reason for doubt is usually given in the columns for $\Delta\lambda$. Thus "triple?" means that the *p*-component is slightly widened and may not be simple, though the widening may be explained by the strength of the component or by the fact that the no-field line is slightly diffuse. "Quadruple?" signifies that the two *n*-components are fairly sharp, but the *p*-component is probably double. The doubtful sextuplet is very common and as a rule has its two *n*-components each widened so that there are probably two pairs, while the *p*-component is either distinctly double or unresolved and considerably widened. The questioned septuplet has almost the same appearance, but shows a central maximum to the widened *p*-component, indicating three constituents. Lines whose *n*-components shade off either outward or toward the center are given in the "Remarks" column as "fringed." The fringes indicate very close, unresolved components. If the structure of the line is very complex, an interrogation point is used without any attempt to give the number of components.

Although the character of separation is thus doubtful in many cases, the large number of plates from which the material was taken gave an opportunity to study each line under various conditions of intensity and degree of separation so that the classification as to character is probably as accurate as can be made without very much higher field-strength combined with as large dispersion as was here used.

In the fourth column, lines are weighted 3, 2, or 1 according as the quality of the components for measurement is good, fair, or poor. Weight 3 indicates that the components are sharply defined, the probable error of measurement being in the third decimal place, often not greater than two or three thousandths of an Ång-

ström. Weight 2 is given to lines which do not permit such close measurement, but which as a rule have a probable error not greater than 10 per cent. Lines weighted 1 are either very faint, much disturbed by blends, or of such complex structure that the components are very diffuse. The error of measurement for such lines may be large, but the figures given show whether the line is to be classed as having small, medium, or large separation. When both the *n*- and the *p*-components are measurable, the weight for each is given, separated by a comma. In case only the *p*-component is measured, a dash before the comma indicates the omission of the weight for the *n*-component.

The fifth and sixth columns give the separation in Ångström units of the *n*- and *p*-components respectively. A single value denotes a pair of components. When there is more than one pair for the same polarization, the largest separation is given first, but the innermost pair is designated as "Pair I." When an odd number of *n*- or *p*-components is present, any outer components that may appear are measured from the central component, the latter being denoted by 0.000. The relative intensities of multiple components are indicated by figures in parentheses after the value of the separation.

The widening of individual components, which often denotes a blend of two or more, is indicated by "w" with subscript 1, 2, or 3, according to the degree of widening. The letter "w" in the *p*-component column shows that the unresolved component may not be single. If a measurement followed by "w" is given in the *n*-component column, a more or less diffuse pair is present, which might be resolved by higher field into two or more pairs.

The letters "n.m." indicate that a separation exists but is not measurable, generally by reason of faintness. In such cases it is usually possible to tell the character of the separation with fair certainty and the line is included on this account. When the components are hazy as well as faint "n.m.w." is used.

The seventh and eighth columns give the values of $\Delta\lambda/\lambda^2$, the positions in the column corresponding to those of the values of $\Delta\lambda$. When λ is expressed in Ångström units, the values given for $\Delta\lambda/\lambda^2$ are to be multiplied by 10^{-8}.

TABLE I

Measurements of Zeeman Effect for Iron

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
3722.071.....	1	Triple	1	0.266	.....	1.920	.....	Faint in spark
3722.729.....	4	12 comps.	2, 2	Pair IV, 0.415 (2)	Pair II, 0.195 (5)	2.994	1.407	
				Pair III, 0.311 (3)	Pair I, n.m. (1)	2.244	?	
				Pair II, 0.211 (3)	.....	1.522	.....	
				Pair I, 0.103 (2)	.....	0.743	.....	
3724.526.....	2	Triple	3	0.256	.....	1.845	.....	
3727.778.....	5	Septuple?	2	0.318 w_1	w_2	2.288	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
3730.534.....	1	Triple?	1	0.341 w_1	w_1	2.451	.....	Blend with faint lines
3731.093.....	1	Triple	1	0.176	.....	1.264	.....	
3732.545.....	2	Triple	3	0.399	.....	2.865	.....	
3733.469.....	3	Quintuple	3, 3	0.157 (2)	0.322	1.127	2.311	
				0.000 (3)	.....	0.000	.....	
				0.158 (2)	.....	1.134	.....	
3735.014.....	10	Triple	2	0.310	.....	2.222	.....	
3735.485.....	1	Quadruple?	2, 2	0.472 w_1	0.166	3.384	1.190	
3737.281.....	7	Septuple?	2	0.254 w_1	w_2	1.818	.....	*n*-comps. fringed. Probably 3 *p*-comps.
3738.454.....	2	Triple	3	0.207	.....	1.481	.....	
3743.508.....	6	Octuple	3, 3	0.208 (4)	0.107 (2)	1.484	0.763	Comps. of faint line to red (computed 3743.615) are superposed on central and outer red *n*-comps., giving apparent dissymmetry. Compare 3788.046
				0.104 (2)	0.000 (3)	0.742	0.000	
				0.000 (2)	0.106 (2)	0.000	0.756	
				0.112 (2)		0.799	.....	
				0.213 (5)	.....	1.520	.....	
3744.251.....	1	Septuple?	1, 1	0.401 w_2	0.283 (1)	2.861	2.018	Probably 4 *n*-comps. Apparently 3 *p*-comps., but faint. Compare 3747
					0.000 (1)	.....	0.000	
					0.415 (1)	.....	2.960	

3745.717.....	5	Septuple?	2	0.228 w_1	w_2	1.624	... :	*n*-comps. fringed. Probably 3 *p*-comps.
3746.058.....	4	Unaffected	..	.....	.....	.....	.....	
3747.065.....	1	Septuple?	1, 1	0.413 w_2	0.306 (1)	2.941	2.180	Apparently same type as 3744. Comps. more distinct
					0.000 (2)	.....	0.000	
					0.341 (1)	.....	2.429	
3748.408.....	4	9 comps.?	2	Pair III, 0.316 (4)	w_3	2.250	.....	Probably 3 *p*-comps. almost resolved
				Pair II, 0.226 (3)	.....	1.609	.....	
				Pair I, 0.101 (1)	.....	0.719	.....	
3749.049.....	1	Quadruple?	2	0.240 w_1	w_2	1.708	.....	
3749.631.....	10	Triple	2	0.289	.....	2.055	.....	
3753.732.....	2	Triple	3	0.395	.....	2.803	.....	
3756.213.....	1	Triple?	2	0.300	w_1	2.126	.....	
3757.081.....	1	Triple?	1	0.197	w_1	1.396	.....	
3757.597.....	1	Triple	1	0.388	.....	2.747	.....	Faint in spark
3758.375.....	8	Triple	3	0.269	.....	1.905	.....	
3760.196.....	2	Triple	3	0.235	.....	1.662	.....	
3760.679.....	1	Quintuple?	3	0.146 (1)	w_2	1.032	.....	*n*-comp. appears as unsymmetrical triplet. No evidence of blend
				0.000 (1)	.....	0.000	.....	
				0.179 (1)	.....	1.265	.....	
3763.945.....	6	Triple	3	0.218	.....	1.538	.....	
3765.689.....	3	Triple	3	0.228	.....	1.608	.....	
3767.341.....	5	Unaffected	..	.....	.....	.....	.....	
3768.173.....	1	Triple	2	0.600	.....	4.225	.....	Faint in spark
3770.446.....	1	Triple	2	0.191	.....	1.343	.....	
3773.803.....	1	Unaffected	..	.....	.....	.....	.....	
3774.971.....	1	Sextuple	2, 1	Pair II, 0.545 (1)	0.240	3.824	1.684	*p*-comps. faint
				Pair I, 0.274 (1)	.....	1.922	.....	
3776.600.....	1	Triple?	2	0.229	w_1	1.605	.....	
3777.593.....	1	Quadruple?	..	n.m.	n.m.	.....	.....	Very faint. Wide separation of *p*-comp.
3778.652.....	1	Triple	1	0.344	.....	2.408	.....	
3779.569.....	1	Triple?	1	0.277	.....	1.938	.....	
3781.330.....	1	?	..	n.m.	w_2	.....	.....	Many comps. *n*-comps. diffuse. Not resolved
3786.092.....	2	Triple	3	0.220	.....	1.535	.....	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
3786.314.....	2	Triple?	..	n.m.w_2	w_2	.....	.....	*n*-comps. diffuse
3786.820.....	2	Unaffected	..	.....	.....	.....	.....	
3788.046.....	4	Octuple	3, 3	0.214 (4)	0.111 (2)	1.491	0.744	Magnetic duplicate of 3743.508
				0.108 (2)	0.000 (3)	0.753	0.000	
				0.000 (1)	0.109 (2)	0.000	0.760	
				0.111 (2)	.....	0.774	.....	
				0.219 (4)	.....	1.526	.....	
4233.772.....	2	9 comps.	2, 2	Pair III, 0.780 (1)	0.140 (2)	4.351	0.780	
				Pair II, 0.550 (2)	0.000 (3)	3.068	0.000	
				Pair I, 0.265 (5)	0.140 (2)	1.478	0.780	
4236.112.....	4	Triple	3	0.452	.....	2.519	.....	
4238.970.....	1	Triple	2	0.320	.....	1.781	.....	
4240.014.....	1	Triple	2	0.440	.....	2.447	.....	
4245.422.....	1	Triple	3	0.493	.....	2.736	.....	
4246.251.....	1	Triple	1	0.273	.....	1.514	.....	
4247.591.....	1	Triple	3	0.356	.....	1.973	.....	
4248.384.....	1	Triple	2	0.377	.....	2.089	.....	
4250.287.....	8	Septuple?	2	0.382 w_1	w_3	2.115	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4250.945.....	9	12 comps.?	2, 2	0.246 w_3	0.211 w_1	1.361	1.168	Probably 8 *n*-, 4 *p*-comps.
4260.640.....	10	Triple	3	0.423	.....	2.330	.....	
4267.122.....	1	Triple	1	0.300	.....	1.648	.....	
4267.985.....	1	Triple	1	0.528	.....	2.899	.....	
4268.915.....	1	Triple?	1	0.462	w_1	2.535	.....	
4271.325.....	9	Triple	3	0.394	.....	2.160	.....	
4271.934.....	10	Triple	3	0.341	.....	1.868	.....	
4282.565.....	1	Septuple?	2	0.310 w_2	w_3	1.691	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4285.605.....	1	Triple	3	0.315	.....	1.715	.....	
4294.301.....	5	Sextuple?	2, 2	0.319 w_2	0.138	1.730	0.748	*n*-comps. almost resolved

4298.195.....	1	Triple?	2	0.457	w_1	2.474	.....	
4299.410.....	5	Triple	3	0.406	.....	2.197	.....	
4302.353.....	1	Triple	2	0.316	.....	1.707	.....	Enhanced line
4303.337.....	1	Sextuple?	2, 2	0.415 w_2	0.265	2.241	1.431	Enhanced line
4305.614.....	1	Quadruple?	1	0.328 w_1	w_2	1.769	.....	
4308.081.....	15	Triple	3	0.320	.....	1.724	.....	
4309.541.....	1	Triple	2	0.325	.....	1.750	.....	
4315.262.....	3	Sextuple?	3, 3	0.517 w_1	0.090	2.777	0.483	
4325.939.....	15	Triple	3	0.245	.....	1.309	.....	
4327.274.....	1	Triple	2	0.313	.....	1.672	.....	
4328.080.....	1	Triple	2	0.246	.....	1.313	.....	
4337.216.....	2	Sextuple?	2, 3	0.264 w_2	0.154	1.404	0.819	
4346.725.....	1	Triple	1	0.292	w_2?	1.545	.....	Blend with air lines
4351.930.....	1	Septuple?	1	0.311 w_2	w_3	1.642	.....	Probably 3 p-comps. May be *Cr*, but given by Lockyer as enhanced line *Fe*
4352.908.....	2	Septuple?	2, 3	0.416 w_2	0.075 (1)	2.195	0.396	n-comps. fringed
					0.000 (2)	.....	0.000	
					0.075 (1)	.....	0.396	
4367.749.....	1	Triple	2	0.311	.....	1.630	.....	
4369.941.....	2	Triple	3	0.282	.....	1.477	.....	
4376.107.....	2	Triple	3	0.424	.....	2.214	.....	
4383.720.....	20	Triple	3	0.332	.....	1.727	.....	
4385.548.....	1	Quadruple	2, 2	0.367	0.391	1.910	2.032	Enhanced line
4388.057.....	1	Triple	..	n.m.	.....	.....	.....	n-comps. blended with adjacent lines
4388.571.....	1	Triple	2	0.432	.....	2.243	.....	
4391.123.....	1	Triple	..	n.m.	.....	.....	.....	Faint
4404.927.....	15	Triple	3	0.334	.....	1.720	.....	
4407.871.....	1	Triple?	2	0.631	w_2	3.247	.....	
4408.582.....	1	Triple?	2	0.488	w_1	2.511	.....	
4415.293.....	10	Septuple?	2	0.338 w_2	w_2	1.734	.....	n-comps. have inner fringes. Probably 3 p-comps.
4422.741.....	1	Sextuple	2, 3	Pair II, 0.432 (1)	0.280	2.208	1.431	
				Pair I, 0.154 (1)	.....	0.787	.....	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
5283.802.....	3	Triple	3	0.623	.....	2.232	.....	
5302.480.....	2	Triple	3	0.632	.....	2.272	.....	
5316.790.....	4	Triple?	2	0.455 w_1	w_2	1.610	.....	Enhanced line. Diffuse comps. may be due to character of line
5324.373.....	5	Triple	3	0.648	.....	2.286	.....	Red comp. slightly stronger than violet
5328.236.....	7	Septuple?	2	0.470 w_1	w_3	1.656	.....	*n*-comps. fringed. Probably 3 *p*-comps.
5328.696.....	3	Sextuple?	2, 2	0.488 w_2	0.275	1.718	0.968	
5340.121.....	2	Triple	2	0.664	.....	2.329	.....	
5341.213.....	3	?	1, 2	0.486 w_3	0.429	1.703	1.502	*n*-comps. blurred, probably at least 6
5353.571.....	1	Triple	..	n.m.	.....	.....	.....	Very faint
5365.069.....	1	Sextuple?	1	0.354 w_2	w_2	1.229	.....	Comps. very diffuse
5365.596.....	1	Triple	1	0.471	.....	1.636	.....	Blend with preceding line
5367.669.....	1	Triple?	2	0.414	w_1	1.437	.....	Diffuse
5370.166.....	2	Triple?	2	0.456	w_1	1.581	.....	Diffuse
5371.734.....	6	9 comps.?	2	0.413 w_2	w_2	1.431	.....	*n*-comps. fringed. Probably 6 *n*-, 3 *p*-comps.
5383.578.....	4	Triple?	3	0.480 w_1	w_1	1.656	.....	
5393.375.....	3	Triple	2	0.673	.....	2.314	.....	
5397.344.....	6	Sextuple?	2, 3	0.630 w_2	0.222	2.163	0.762	
5404.357.....	5	Triple?	2	0.467 w_1	w_1	1.599	.....	Diffuse
5405.989.....	6	9 comps.?	2, 2	Pair II, 0.461 (1)	0.117 (2)	1.577	0.400	Probably third pair *n*-comps. outside
				Pair I, 0.222 (4)	0.000 (3)	0.760	0.000	
					0.121 (2)	.....	0.414	
5411.124.....	3	Triple?	2	0.435 w_1	w_1	1.486	.....	Diffuse
5415.416.....	5	Triple?	2	0.510 w_1	w_1	1.739	.....	Diffuse
5424.290.....	5	Triple?	2	0.498 w_1	w_1	1.693	.....	Diffuse

5429.911.....	6	10 comps.?	2, 3	0.607 w_2	0.300	2.059	1.017	3 or possibly 4 pairs *n*-comps. Probably weak inner pair *p*-comps.
5434.740.....	5	Unaffected	..	.....	.....	.....	.....	
5445.259.....	2	Triple?	2	0.415	w_1	1.399	.....	Diffuse
5447.130.....	5	12 comps.	2, 3	Pair IV, 0.874 (1)	Pair II, 0.477 (6)	2.946	1.507	*n*-comps. barely resolved
				Pair III, 0.701 (2)	Pair I, 0.226 (1)	2.363	0.762	
				Pair II, 0.477 (2)	.....	1.608	.....	
				Pair I, 0.219 (1)	.....	0.738	.....	
5455.834.....	4	Quintuple	2, 3	0.347 (1)	0.680	1.163	2.283	Central line of *n*-triplet displaced 0.009 Å toward red from normal
				0.000 (2)	.....	0.000	.....	
				0.345 (1)	.....	1.160	.....	
6136.829.....	5	Triple	3	0.515	.....	1.367	.....	
6137.915.....	5	Triple	3	0.654	.....	1.736	.....	
6141.938.....	1	Triple?	2	1.017	w_1	2.696	.....	Faint
6148.040.....	1	Sextuple?	1	0.649 w_2	w_1	1.717	.....	Very diffuse
6149.458.....	1	Quadruple	1, 1	0.812	0.740	2.148	1.950	Enhanced line
6157.945.....	1	Triple?	2	0.719	w_1	1.896	.....	
6165.577.....	1	Sextuple?	..	n.m.w_2	w_2	.....	.....	*n*-comps. diffuse
6170.730.....	1	Sextuple?	2	0.725 w_2	w_3	1.904	.....	*p*-comp. almost resolved
6173.553.....	1	Triple	2	1.590	.....	4.171	.....	Difficult. Blend with air band
6180.420.....	1	Triple?	..	n.m.	w_1	.....	.....	Faint
6188.210.....	1	Triple	..	n.m.	.....	.....	.....	Faint
6191.779.....	5	Triple	3	0.541	.....	1.411	.....	
6200.527.....	1	Sextuple?	1	1.026 w_2	w_3	2.669	.....	Comps. diffuse
6213.644.....	1	Sextuple	2, 3	Pair II, 1.603 (1)	0.572	4.151	1.481	
				Pair I, 0.798 (1)	.....	2.067	.....	
6215.360.....	1	Triple	2	0.583	.....	1.509	.....	
6219.494.....	1	Sextuple?	2, 2	0.991 w_2	0.385	2.562	0.995	
6230.943.....	6	Triple	3	0.767	.....	1.976	.....	
6232.856.....	1	Sextuple?	2	1.205 w_1	w_2	3.102	.....	
6238.598.....	1	Sextuple?	1, 1	1.042 w_1	0.512	2.677	1.315	Enhanced line
6246.535.....	2	Sextuple?	2, 2	0.958 w_1	0.278	2.456	0.713	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
6247.774.....	1	Septuple?	1	0.670 w_2	w_3	1.716	.....	Enhanced line. *n*-comps. very diffuse. Probably 3 *p*-comps.
6252.773.....	3	Triple	3	0.582	.....	1.488	.....	
6254.456.....	1	Triple	3	0.952	.....	2.434	.....	
6256.572.....	1	Sextuple?	–, 2	n.m.w_3	0.696	.....	1.778	*n*-comps. very diffuse
6265.348.....	1	Sextuple?	2, 2	0.969 w_2	0.278	2.469	0.708	
6270.442.....	1	Triple?	..	n.m.	w_1	.....	.....	Very faint
6291.184.....	1	Sextuple?	1	1.014	w_2	2.562	.....	
6298.007.....	1	Sextuple?	..	n.m.w_2	w_3	.....	.....	
6301.718.....	3	Sextuple?	2, 2	1.063 w_2	0.392	2.676	0.987	*n*-comps. probably double
6302.709.....	1	Triple	2	1.618	.....	4.073	.....	
6315.517.....	1	Sextuple?	1	0.906 w_1	w_2	2.271	.....	Faint. *p*-comp. almost resolved. Enhanced line
6318.239.....	3	Triple	2	0.452	.....	1.132	.....	
6322.907.....	1	Triple	2	0.965	.....	2.414	.....	
6331.067.....	1	Sextuple?	1	0.820 w_2	w_1	2.046	.....	Very faint
6335.554.....	3	Septuple?	2	0.665 w_2	w_3	1.656	.....	Probably 3 *p*-comps.
6337.048.....	3	Sextuple	2, 2	Pair II, 1.641 (1)	0.632	4.086	1.574	
				Pair I, 0.946 (1)	.....	2.356	.....	
6339.096.....	1	Triple?	..	n.m.	.....	.....	.....	Very faint
6344.371.....	1	Sextuple?	1	0.771 w_1	n.m.	1.916	.....	Very faint. *p*-comp. double
6355.246.....	1	Triple	1	0.730	.....	1.808	.....	Very faint
6358.898.....	1	Sextuple?	1	0.746 w_2	w_1	1.845	.....	Very faint
6380.958.....	1	Triple	1	0.464	.....	1.140	.....	Enhanced line
6393.820.....	8	Triple	2	0.593	.....	1.450	.....	
6400.217.....	9	Septuple?	2	0.802 w_1	w_2	1.958	.....	*n*-comps. slightly fringed. Probably 3 close *p*-comps.

6408.233.....	1	Septuple?	–, 2	n.m.	0.343 (1) 0.000 (2) 0.349 (1)	....	0.837 0.000 0.852	Apparently 4 weak *n*-comps. about equally spaced
6411.865.....	5	Septuple?	2	0.686 w_2	w_3	1.668	.....	*n*-comps. fringed, probably 3 *p*-comps.
6417.133.....	1	?	1	1.018 w_2	w_3	2.472	.....	*n*- and *p*-comps. very diffuse. Enhanced line
6420.169.....	1	Sextuple?	1	0.743 w_2	w_2	1.802	.....	*n*-comps. diffuse. Enhanced line
6421.570.....	2	Triple	3	0.993	.....	2.408	.....	
6431.066.....	3	Sextuple?	2	0.775 w_1	w_1	1.873	.....	

TABLE II

MEASUREMENTS OF ZEEMAN EFFECT FOR TITANIUM

λ	INTENSITY	CHARACTER OF SEPARATION	WEIGHT	Δλ		Δλ/λ²		REMARKS
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
4278.390.....	2	Triple	3	0.304	.....	1.661	.....	
4281.530.....	2	Octuple	3, 3	0.444 (4)	0.222 (2)	2.422	1.211	
				0.218 (2)	0.000 (3)	1.189	0.000	
				0.000 (1)	0.224 (2)	0.000	1.222	
				0.218 (2)	.....	0.189	.....	
				0.448 (4)	.....	2.443	.....	
4282.860.....	3	Triple	3	0.244	.....	1.330	.....	
4285.164.....	5	Quadruple?	3	0.566	w_1	3.083	.....	
4286.168.....	4	Sextuple?	2, 3	0.400 w_2	0.166	2.177	0.904	
4287.566.....	4	Sextuple?	2, 3	0.421 w_2	0.146	2.290	0.794	
4288.038.....	2	Septuple?	2	0.516 w_2	w_3	2.806	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
4289.237.....	4	12 comps.	2, 3	Pair IV, 0.586 (1)	Pair II, 0.286 (6)	3.186	1.566	
				Pair III, 0.443 (2)	Pair I, 0.140 (1)	2.408	0.761	
				Pair II, 0.306 (2)	.....	1.663	.....	
				Pair I, 0.144 (1)	.....	0.783	.....	
4290.377.....	10	?	2	0.284 w_2	w_3	1.543	.....	*n*-comps. strongly fringed 3 or more *p*-comps. Enhanced line
4291.114.....	2	Quintuple	3, 3	0.221 (1)	0.445	1.200	2.417	
				0.000 (2)	.....	0.000	.....	
				0.220 (1)	.....	1.195	.....	
4291.375.....	2	Triple	1	0.230	.....	1.249	.....	Difficult blend with 4291.114
4294.204.....	10	Triple	3	0.361	.....	1.958	.....	Enhanced line
4295.014.....	4	Unaffected	..	.....	.....	.....	.....	
4298.828.....	4	Septuple	2, 3	Pair II, 0.292 (1)	0.060 (2)	1.580	0.325	*p*-comps. distinctly unsymmetrical
				Pair I, 0.145 (5)	0.000 (3)	0.784	0.000	
					0.086 (2)	.....	0.465	

4299.410.....	3	Quadruple?	1	0.430	w_2	2.327	.....	
4299.803.....	2	Triple?	1	0.356	w_1	1.925	.....	
4300.211.....	8	?	2	0.367 w_1	w_2	1.985	.....	*n*-comps. fringed. 3 or more *p*-comps. Enhanced line
4300.732.....	2	Septuple?	2	0.265 w_1	w_2	1.432	.....	*n*-comps. fringed, probably 3 *p*-comps.
4301.158.....	3	Triple?	3	0.350	w_1	1.892	.....	
4302.085.....	5	Sextuple	3, 3	Pair II, 0.585 (1)	0.216	3.161	1.167	Enhanced line
				Pair I, 0.151 (3)	.....	0.816	.....	
4306.078.....	8	Septuple?	2	0.367 w_1	w_2	1.979	.....	*n*-comps. fringed, probably 3 *p*-comps.
4308.081.....	8	Octuple	2, 2	Pair III, 0.588 (1)	0.236	3.168	1.272	Blend with iron impurity line probably gives apparent dissymmetry in intensity of *n*-comps. Enhanced line
				Pair II, 0.442 (2)	.....	2.382	.....	
				Pair I, 0.291 (3)	.....	1.568	.....	
4311.880.....	1	Sextuple?	2	0.147	w_1	0.791	.....	Outer pair *n*-comps. not measurable. Possibly 3 *p*-comps. Blend with faint lines
4313.034.....	8	Sextuple?	2, 2	0.449 w_2	0.159	2.414	0.855	Enhanced line
4314.964.....	3	Triple	3	0.424	.....	2.277	.....	
4315.138.....	5	Quadruple	3, 3	0.392	0.349	2.105	1.874	Enhanced line
4316.962.....	2	Triple	3	0.207	.....	1.111	.....	Enhanced line
4318.817.....	3	Triple	3	0.337	.....	1.807	.....	
4321.119.....	3	Sextuple	3, 3	Pair II, 0.785 (2)	0.261	4.204	1.398	Enhanced line
				Pair I, 0.257 (1)	.....	1.376	.....	
4321.813.....	3	Triple	3	0.310	.....	1.660	.....	
4323.531.....	1	Triple	2	0.464	.....	2.482	.....	
4325.306.....	3	Triple	3	0.301	.....	1.609	.....	
4326.520.....	2	Triple	3	0.403	.....	2.152	.....	
4330.405.....	3	Sextuple?	2	0.415 w_1	w_2	2.213	.....	Enhanced line
4330.866.....	3	?	2	0.654 w_2	w_3	3.487	.....	Enhanced line. *n*-comps. have inner fringes
4338.084.....	10	Triple	3	0.247	.....	1.312	.....	Enhanced line

TABLE II—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
4468.663.....	15	Triple	3	0.340	.....	1.702	.....	Enhanced line
4469.316.....	1	Triple	2	0.426	.....	2.133	.....	
4471.017.....	2	10 comps.?	2, 2	Pair III, 0.724 (1)	0.458	3.622	2.292	Trace of inner pair *p*-comps.
				Pair II, 0.386 (1)	.....	1.931	.....	
				Pair I, 0.126 (1)	.....	0.630	.....	
4471.408.....	2	9 comps.	2, 2	Pair III, 0.826 (1)	0.113 (2)	4.132	0.565	
				Pair II, 0.613 (2)	0.000 (3)	3.066	0.000	
				Pair I, 0.364 (4)	0.116 (2)	1.821	0.580	
4475.026.....	2	Septuple?	2	0.509 w_1	w_3	2.542	.....	Probably 4 *n*-, 3 *p*-comps.
4479.879.....	2	Triple	3	0.829	.....	4.130	.....	
4480.752.....	1	Triple	2	0.611	.....	3.043	.....	
4481.438.....	3	Triple	3	0.548	.....	2.729	.....	
4482.904.....	2	Sextuple?	2	0.498 w_1	w_2	2.478	.....	
4488.493.....	6	Triple	3	0.355	.....	1.762	.....	Enhanced line
4489.262.....	3	9 comps.	2, 2	Pair III, 0.858 (1)	0.111 (2)	4.258	0.551	Compare λ 4471.408
				Pair II, 0.597 (2)	0.000 (3)	2.963	0.000	
				Pair I, 0.382 (4)	0.110 (2)	1.896	0.546	
4495.182.....	1	Triple	1	0.353	.....	1.747	.....	
4496.318.....	3	Triple	3	0.493	.....	2.439	.....	
4497.842.....	1	Triple	2	0.534	.....	2.639	.....	
4501.445.....	15	Triple	3	0.298	.....	1.471	.....	Enhanced line
4512.906.....	4	Triple	3	0.501	.....	2.460	.....	
4518.198.....	4	Quadruple?	3	0.498	w_1	2.440	.....	
4518.866.....	1	Triple	2	0.220	.....	1.077	.....	
4522.974.....	4	Sextuple?	3	0.502 w_1	w_2	2.454	.....	
4527.490.....	4	Octuple	3, 3	0.324 (7)	0.164 (1)	1.581	0.800	
				0.162 (2)	0.000 (2)	0.790	0.000	
				0.000 (1)	0.165 (1)	0.000	0.805	
				0.166 (4)	.....	0.810	.....	
				0.338 (7)	.....	1.649	.....	
4529.656.....	2	Sextuple?	2, 2	0.358 w_2	0.278 w_1	1.745	1.355	Enhanced line

4533·419.....	5	Triple	3	0.469	.....	2.282	.....	
4534·139.....	6	Triple?	2	0.360 w_1	w_1	1.751	.....	*n*-comps. slightly fringed. Enhanced line
4534·953.....	4	Triple	3	0.449	.....	2.183	.....	
4535·741.....	3	Triple	3	0.424	.....	2.061	.....	
4536·094.....	3	Triple	2	0.323	.....	1.570	.....	
4536·222.....	3	Unaffected?	..	.....	.....	.....	.....	No resolution. Blend with 36.094 may conceal slight widening of *n*-comp.
4537·389.....	1	Triple	1	0.355	.....	1.725	.....	
4544·190.....	1	Quadruple?	1	0.308	w_2	1.492	.....	
4544·864.....	3	Octuple	3, 3	0.334 (7)	0.171 (1)	1.617	0.828	Comps. in all respects similar to 4427.490
				0.168 (2)	0.000 (2)	0.813	0.000	
				0.000 (1)	0.166 (1)	0.000	0.804	
				0.170 (4)	.....	0.823	.....	
				0.332 (7)	.....	1.607	.....	
4548·938.....	3	Septuple?	2	0.560 w_1	w_2	2.706	.....	*n* comps. have inner fringes. Probably 3 *p*-comps.
4549·808.....	20	Triple	3	0.440	.....	2.125	.....	Enhanced line
5016·340.....	7	Sextuple?	2, 3	0.543	0.214	2.158	0.851	
5020·208.....	8	Octuple?	2, 3	0.507 w_3	0.276	2.012	1.095	Probably 3 pairs *n*-comps.
5023·052.....	8	12 comps.?	2, 3	0.466 w_3	0.370 w_3	1.847	1.466	Probably 4 pairs *n*-comps. *p*-comps. have inner fringes.
5025·027.....	7	10 comps.	2, 2	Pair III, 0.684 (1)	Pair II, 0.546 (6)	2.709	2.162	
				Pair II, 0.416 (2)	Pair I, 0.269 (1)	1.647	1.065	
				Pair I, 0.133 (4)	.....	0.527	.....	
5025·749.....	5	Triple	3	0.471	.....	1.865	.....	
5036·089.....	10	Triple	3	0.455	.....	1.794	.....	
5036·645.....	8	Triple	3	0.436	.....	1.718	.....	
5038·579.....	8	Triple	3	0.340	.....	1.339	.....	
5040·138.....	8	Triple	3	0.404	.....	1.590	.....	
5053·056.....	3	Triple?	2	0.449	w_1	1.759	.....	

TABLE II—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
5062.285.....	3	Triple?	2	0.412	w_1	1.608	.....	
5064.244.....	1	Triple	..	n.m.	.....	.....	.....	Very faint
5064.836.....	8	Triple	3	0.463	.....	1.805	.....	
5066.174.....	1	Sextuple?	−,1	n.m.w_3	0.407	.....	1.586	
5069.592.....	1	Triple	2	0.235	.....	0.914	.....	
5071.666.....	4	Sextuple?	1, 1	0.470 w_2	0.275	1.827	1.069	
5072.479.....	6	Triple	3	0.502	.....	1.951	.....	Enhanced line
5087.239.....	4	Triple	3	0.329	.....	1.271	.....	
5113.617.....	5	Triple	3	0.431	.....	1.648	.....	
5120.592.....	7	Triple	3	0.434	.....	1.655	.....	
5129.336.....	8	Triple	3	0.478	.....	1.812	.....	Enhanced line
5145.636.....	6	Triple	3	0.493	.....	1.862	.....	
5147.652.....	5	Septuple?	2	0.805 w_2	w_3	3.038	.....	Probably more than 7 comps. *p*-comps. almost resolved
5152.361.....	5	Quadruple?	3	0.671 w_1	w_1	2.528	.....	
5154.244.....	4	Sextuple?	2	0.666 w_2	w_2	2.507	.....	Enhanced line
5173.917.....	10	Triple	2	0.292	.....	1.091	.....	
5186.073.....	8	Triple	3	0.385	.....	1.432	.....	Enhanced line
5188.863.....	12	Triple	3	0.512	.....	1.902	.....	Enhanced line
5193.139.....	10	Triple	3	0.468	.....	1.735	.....	
5201.260.....	3	Triple?	2	0.648 w_1	w_1	1.396	.....	
5206.215.....	4	Triple	2	0.470	.....	1.734	.....	
5210.555.....	10	Triple	3	0.547	.....	2.014	.....	
5210.875.....	4	Sextuple?	2	0.326 (2)	w_2	1.191	.....	Unsymmetrical. Probably 4 *n*-comps., 2 violet blended, 2 red separated. *p*-comp. displaced 0.062 to violet from normal. All comps. measured from normal
				normal	.....	.....	.....	
				0.264 (1)	.....	0.969	.....	
				0.400 (1)	.....	1.468	.....	
5222.849.....	3	Unaffected	..	.....	.....	.....	.....	

5866.675.....	10	Triple	3	0.674	.....	1.958	.....	
5880.490.....	3	Triple	3	0.830	.....	2.401	.....	
5899.518.....	7	Triple	3	0.656	.....	1.885	.....	
5903.555.....	2	Triple	2	0.876	.....	2.513	.....	Red comp. strongest?
5918.773.....	3	Triple	3	0.882	.....	2.518	.....	
5922.334.....	5	Triple	2	0.312	.....	0.890	.....	
5938.035.....	2	Sextuple?	2, 1	0.840 w_2	0.226	2.382	0.641	*p*-comp. scarcely resolved
5941.985.....	5	Sextuple	2, 2	Pair II, 0.905 (2)	0.547	2.563	1.549	
				Pair I, 0.295 (3)	.....	0.836	.....	
5953.386.....	8	Triple	3	0.637	.....	1.798	.....	
5966.055.....	7	Triple	3	0.590	.....	1.617	.....	
5978.768.....	7	Triple	3	0.533	.....	1.491	.....	
5999.920.....	2	Quadruple?	2	0.823 w_1	w_2	2.286	.....	
6064.853.....	2	Triple	3	1.159	.....	3.151	.....	
6085.470.....	4	Sextuple?	2, 2	1.001 w_2	0.307	2.703	0.829	
6091.395.....	6	Triple	3	0.679	.....	1.830	.....	
6093.030.....	1	Triple	2	0.755	.....	2.034	.....	
6098.870.....	2	Triple?	2	0.563 w_1	w_1	1.514	.....	
6121.215.....	1	Triple	2	0.622	.....	1.660	.....	
6126.435.....	5	Septuple?	2	0.706 w_1	w_3	1.881	.....	*n*-comps. fringed. Probably 3 *p*-comps.
6146.445.....	1	Triple	2	0.476	.....	1.260	.....	
6149.950.....	1	Triple	2	0.698	.....	1.845	.....	
6215.630.....	3	Triple	3	0.697	.....	1.804	.....	
6220.700.....	2	Triple	2	0.618	.....	1.597	.....	
6221.552.....	1	Triple	2	0.511	.....	1.320	.....	
6258.322.....	10	Triple	2	0.661	.....	1.688	.....	Red *n*-comps. of first line blended with violet comp. of second
6258.927.....	12	Triple	2	0.714	.....	1.823	.....	
6261.316.....	10	Triple	3	0.556	.....	1.418	.....	
6303.985.....	5	Octuple?	2, 2	0.565 w_3	0.554	1.422	1.394	*n*-comps. very wide and diffuse. Probably at least 3 pairs
6312.456.....	5	Octuple?	2, 2	0.766 w_3	0.463 w_1	1.923	1.162	*n*-comps. wide, not so diffuse as 6304. Probably 3 pairs. Possibly 4 *p*-comps.

EXPLANATION OF THE PLATES

Enlargements of selected portions of the iron and titanium spectra are reproduced in Plates X–XIII. In Plate X, the spectra of the spark without magnetic field is placed above and below, while the three magnetic-field spectra are those given when the light is observed across the lines of force with a Nicol in two positions 90° apart, and without a Nicol, the first two giving the *p*- and *n*-components respectively, while the last gives the two superposed.

For the other plates, the same plan is followed, but only the spectra showing the *p*- and *n*-components are presented. The four spectra in Plate XII show two portions of the titanium spectrum, the range of intensity among the lines being so great that one photograph can do justice only to a certain set.

The number of lines for each type of separation, including both the clear and the questioned cases as given in the complete tables, is shown in Table III.

The publication discusses in detail the characteristics of each type.

RELATION OF SEPARATIONS TO THE NORMAL INTERVAL

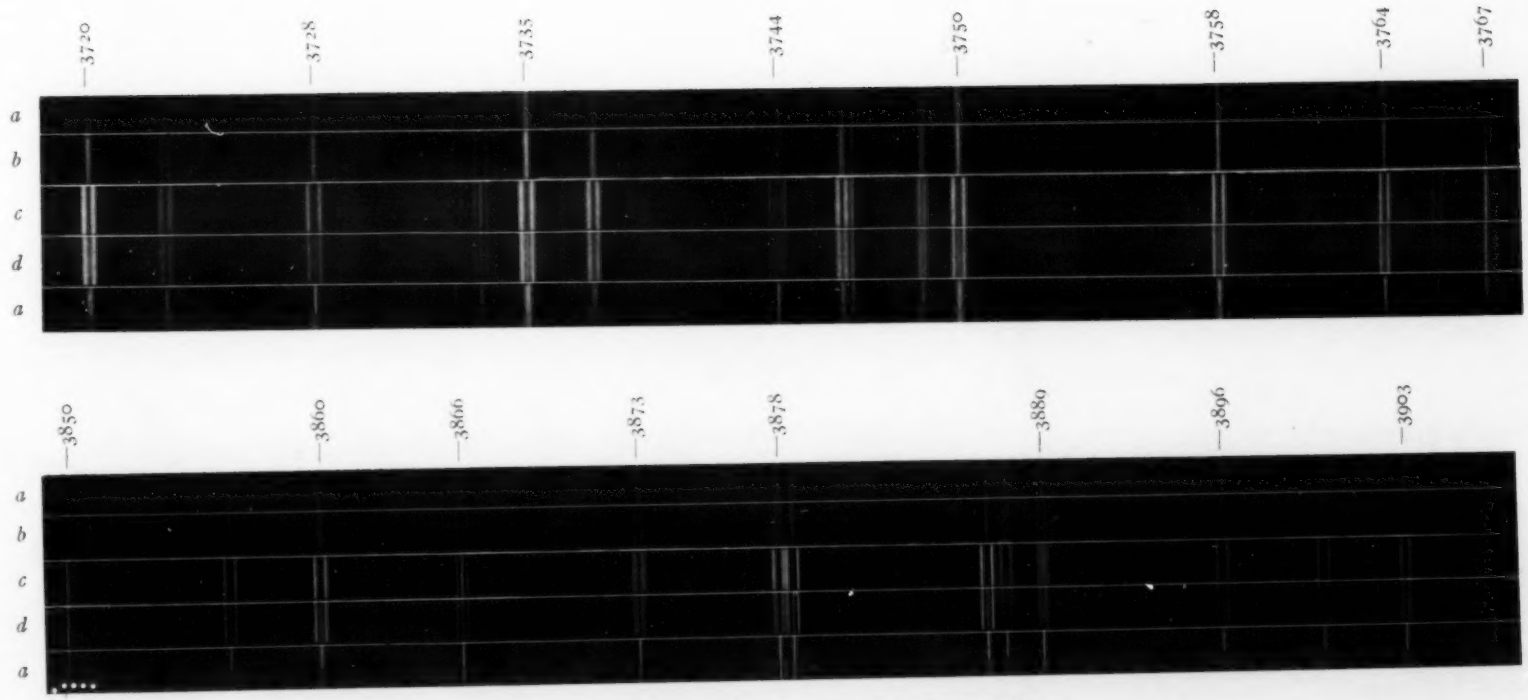
The study of how generally the separations observed show a simple relation to a fundamental interval has been gone into with some detail. The relation

$$a=\frac{\Delta\lambda}{\lambda^2}=\frac{e}{m}\cdot\frac{H}{4\pi v},$$

where e/m is the ratio of charge to mass of the electron, H the field-strength, and v the velocity of light, gives values for a of 0.753 and 0.812 for the field-strengths corresponding to the measurements for iron and titanium, respectively. The "normal triplet" should then show values of $\Delta\lambda/\lambda^2$ for the distance between the side components of about 1.500 and 1.600, respectively.

Various types of separation are considered in turn with regard to how nearly the measured separations can be expressed in terms of the normal interval a. In the iron spectrum, 272 clear triplets were measured, i.e., triplets whose components show no widening which would indicate that the line is of higher type. Of these, 230

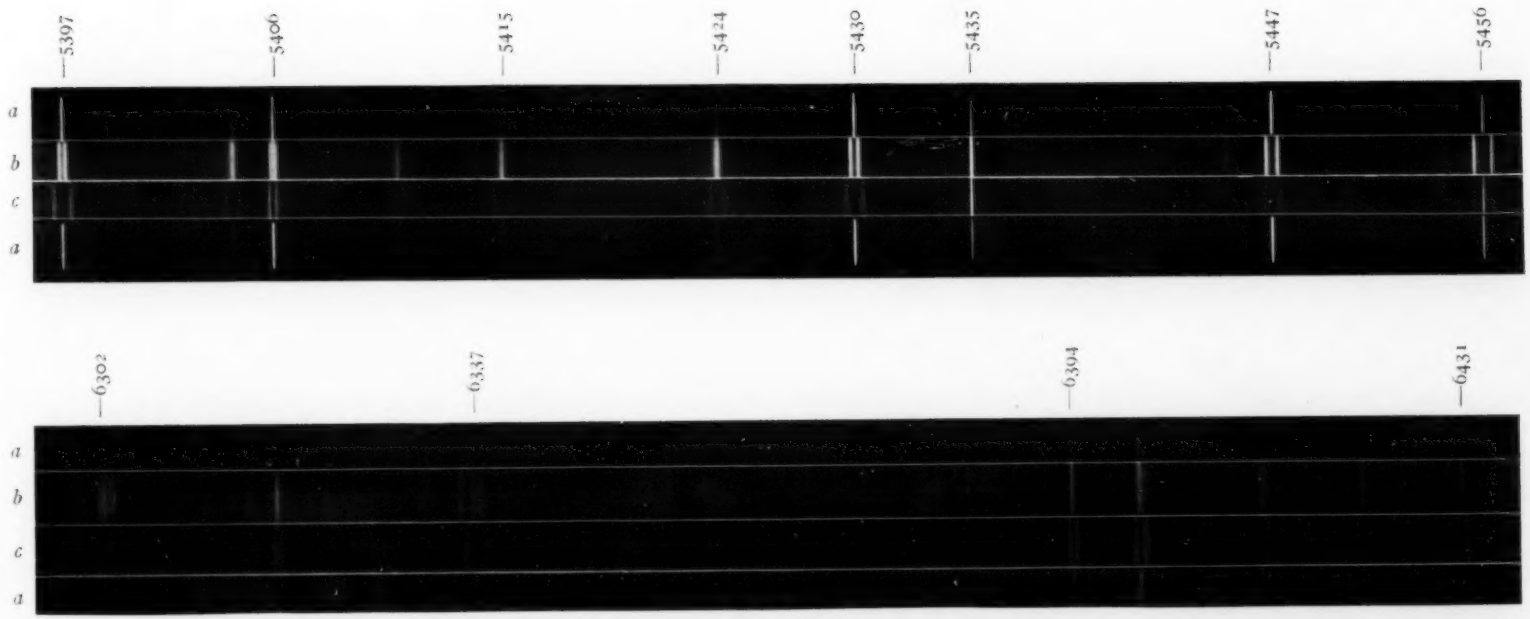
PLATE X



EFFECT OF MAGNETIC FIELD ON THE SPECTRUM OF IRON

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components. *d*. Spectrum without Nicol, *n*- and *p*-Components superposed

PLATE XI

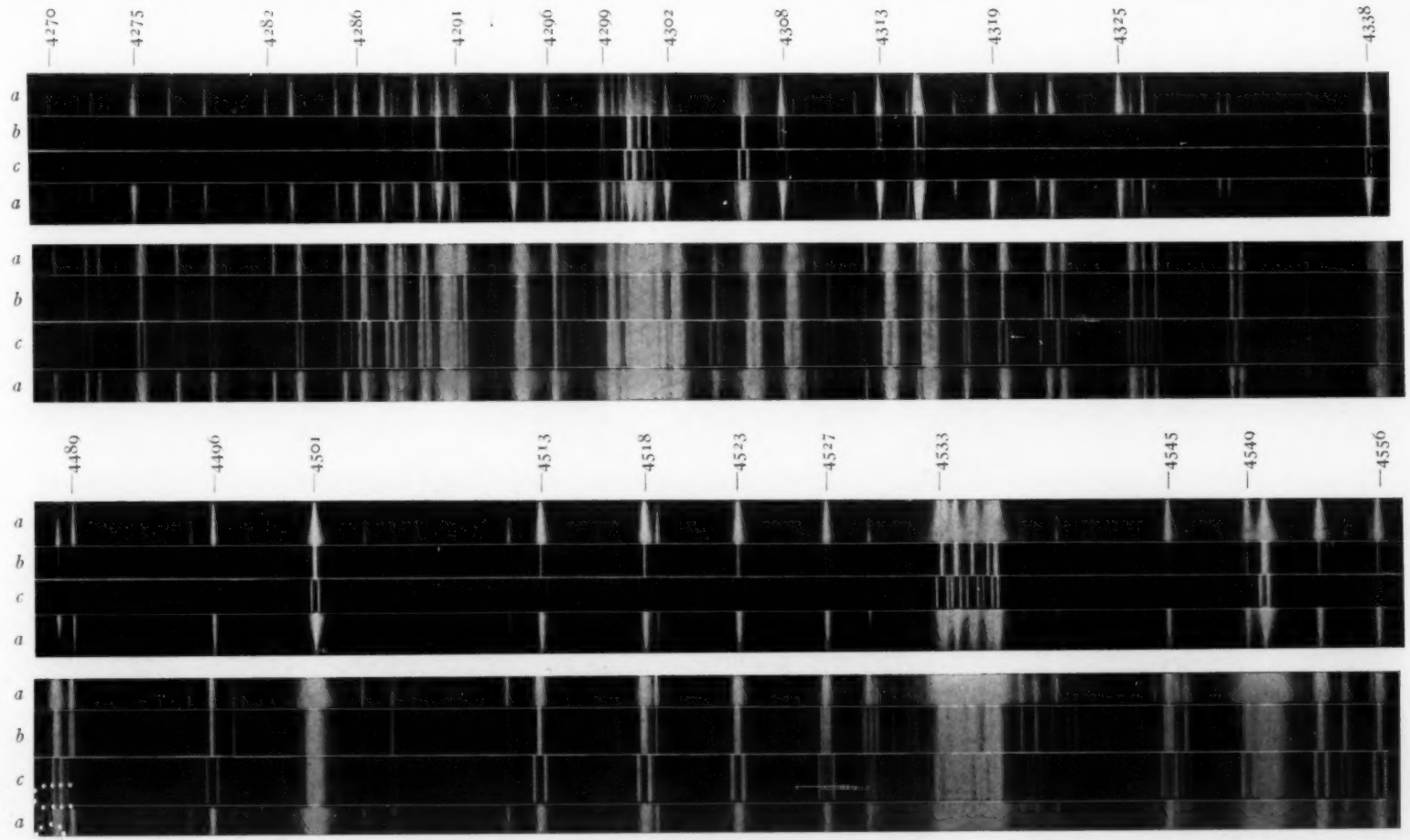


EFFECT OF MAGNETIC FIELD ON THE SPECTRUM OF IRON

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

PLATE XII

PLATE XII

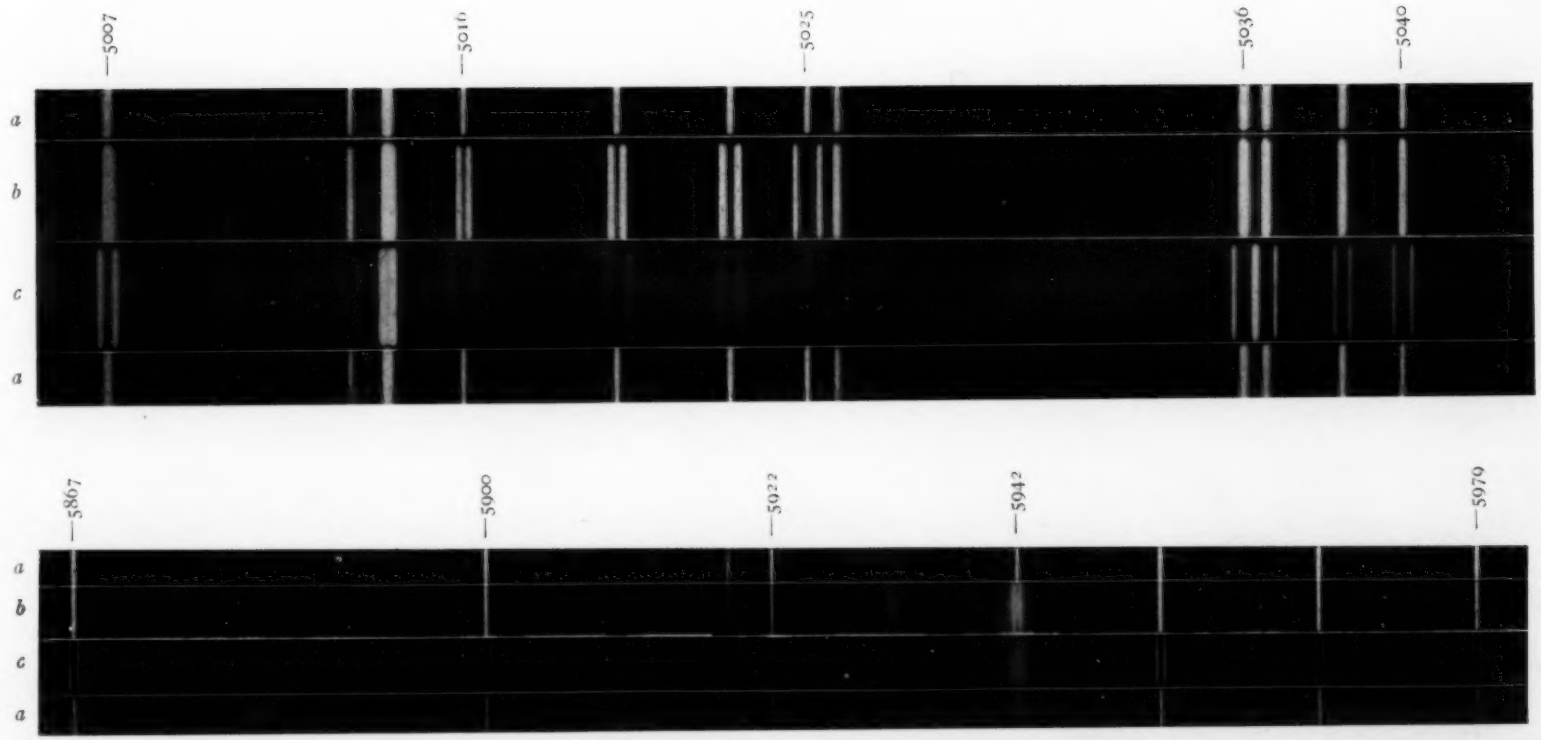


EFFECT OF MAGNETIC FIELD ON THE SPECTRUM OF TITANIUM

Experimental conditions varied to bring out both strong and weak lines

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

PLATE XIII



EFFECT OF MAGNETIC FIELD ON THE SPECTRUM OF TITANIUM

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

gave separations which could be expressed readily as multiples of $a/2$, 54 showing a value of $\Delta\lambda/\lambda^2$ close to $2a$, and 72 close to $3a$. The titanium table contains 242 clear triplets, 145 of which gave multiples of $a/2$, 73 of these being normal triplets. Many of the remaining triplets allow their separations to be expressed as even multiples of $a/4$, but, with the fields employed, this brings the unit close to the possible error of measurement, except for lines of high weight.

TABLE III

SUMMARY OF TYPES OF SEPARATION

Separation	Iron	Titanium
Unaffected	9	4
Triple	393	291
Quadruple	49	28
Quintuple	7	5
Sextuple	118	77
Septuple	37	12
Octuple	6	11
9 components	9	3
10 components	7	7
11 components	2	2
12 components	4	2
13 components	2	0
Unclassified	19	16
Total	662	458

The lines appearing as quadruplets (in many cases really sextuple owing to two pairs of n-components being blended) in the majority of cases allow their separations to be expressed as multiples of a or $a/2$, the most common ratios of n- and p-separations being $2a:a$ and $3a:a$.

The lines of more complex type which are fully resolved, ranging from quintuplets to 12-component lines, are tabulated in a form to show how the separations may be expressed in terms of the normal interval and also to give the ratio, usually very simple, between the successive separations. Fifty-seven complex lines are thus treated, and in almost every case the ease with which the separations can be expressed in terms of a leaves no doubt as to the generality of a relation of this sort. As an example, the 12-component lines $\lambda\lambda$ 3722.729, 3872.639, and 5447.130 of iron, and 4289.237 of titanium, some of which are given in Tables I and II

in this abstract, present in each case the following arrangement of the six pairs of components:

$\pm 2a$	n	4
$3a/2$	n	3
a	n,p	2
$a/2$	n,p	1

The ratio separations for successive pairs is thus 1:2:3:4. Not all lines show so simple a ratio, and probably a much stronger field, permitting the use of small fractional parts of a, would cause some revision of the relations here given for a few lines. The results in general, however, are fully in accordance with the rule given by Runge[1] that for complex lines the distances of the various components from the center may be expressed as multiples of aliquot parts of a.

This treatment of complex separations has brought to notice many cases of "magnetic duplicates," lines showing exactly the same structure and with the same intervals between components. The knowledge of the structure of these lines may be of service in the search for series relations, but as yet nothing definite in this direction has been developed.

DISSYMMETRY

The number of lines showing clear dissymmetry in spacing or intensity of components is not large, fourteen such lines being listed for the two spectra.

Measurements were made for a number of sharp iron triplets to test the dissymmetry predicted by Voigt and investigated by Zeeman and others, according to which the violet component of a triplet should have a slightly greater spacing from the central component, while the red component should be slightly stronger than the violet. Of 26 lines favorable for this purpose, all but 3 show a larger spacing for the violet component, while most of the lines show a difference considerably greater than the probable error of measurement. Numerous triplets also appear for which the red component is very slightly stronger than the violet. A detailed study must wait until a much greater field-strength is available,

[1] *Physikalische Zeitschrift*, **8**, 232, 1907.

but this preliminary examination indicates that the effect is real for many triplets.

LAW OF CHANGE OF THE AVERAGE SEPARATION OF THE *n*-COMPONENTS WITH THE WAVE-LENGTH

The extensive range of wave-length studied for iron and titanium, together with the large number of lines measured, has furnished material for a statistical study of the change of the average separation with the wave-length. The method of treatment has been to obtain the mean value of $\Delta\lambda/\lambda^2$ for the *n*-components of lines for each 500 Å from λ 3700 to λ 6700. When there are two or more pairs of *n*-components the mean of the separations is taken. The averages thus obtained are presented in Table IV. The means for the six groups of 500 Å are given first, then the means for the three groups of 1000 Å. These latter are the means for the whole number of lines included in the range, not the average of the means for the 500-groups.

TABLE IV

MEANS OF $\Delta\lambda/\lambda^2$ (*n*-COMPONENTS) FOR SUCCESSIVE REGIONS OF WAVE-LENGTH

RANGE OF λ	IRON		TITANIUM	
	No. Lines	Mean $\Delta\lambda/\lambda^2$	No. Lines	Mean $\Delta\lambda/\lambda^2$
3700–4200....	267	2.003	80	1.909
4200–4700....	101	2.051	152	2.027
4700–5200....	74	2.125	81	1.684
5200–5700....	62	1.932	47	1.819
5700–6200....	37	1.837	34	1.942
6200–6700....	41	2.131	28	1.764
3700–4700....	368	2.016	232	1.986
4700–5700....	136	2.037	128	1.734
5700–6700....	78	1.989	62	1.862

The close agreement of the means shows that there is an approximate constancy of the values of $\Delta\lambda/\lambda^2$ for different parts of the spectrum. Taking the successive means of the 500-groups, the average value for iron is 2.013, for titanium 1.858. The largest deviation from the mean for any group is 8.7 per cent for iron and 9.4 per cent for titanium. For neither element is there any systematic change of the means for successive groups.

The means for the groups of 1000 Å show a still closer agreement, the largest deviation from the mean of these groups being only 1.2 per cent for iron and 6.8 per cent for titanium.

The mean values for titanium run smaller than those for iron, although the titanium measurements correspond to the greater field-strength. When a number of spectra are examined in this way, it may prove that this difference is connected with certain properties of the elements concerned.

This constancy of the mean values of $\Delta\lambda/\lambda^2$ appears to be based on the general relation of the separations to the fundamental interval a, and to result from the fact that the great majority of the separations for the n-components range from the values of $2a$ to $3a$. Among triplet lines, the greater number show a separation larger than $2a$. The exceptionally large and small values for triplets, together with the mean separations of the complex lines, combine to form a fairly definite mean which holds for the whole range of spectrum examined.

Since $\Delta\lambda/\lambda^2$ is thus shown to be nearly constant, it may be said that *for the spectra of iron and titanium and probably for spectra in general, the mean separation of the n-components varies as the square of the wave-length.* A similar rule must hold for the p-components, since complex lines of the same structure in different parts of the spectrum show the same relation to the interval a.

A result obtained by a different method, but doubtless based on the same general relation, was given by Mr. Hale[1] in a comparison of sun-spot doublets with Zeeman separations on laboratory plates. He observed that the mean $\Delta\lambda$ divided by the square of the mean λ agreed exactly for two regions of the iron spectrum.

THE EFFECT OF THE MAGNETIC FIELD UPON ENHANCED LINES

In my former paper on the titanium spectrum, as well as in the recent papers of Mr. Babcock[2] on the Zeeman effect for chromium and vanadium, the conclusion was reached that the enhanced lines as a class are not affected differently from the non-enhanced lines

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, Nos. 52 and 55; *Astrophysical Journal*, **33**, 217, 1911.

of these elements. The larger amount of material in the present publication has verified this conclusion in general, but it also presents evidence that the stronger enhanced lines of titanium show a tendency toward one type. The question is of interest because the enhanced lines form a fairly well defined class, requiring certain physical conditions of the light-source for their production, and a modification of these conditions affects the intensity and appearance of all such lines in the same way though in different degrees.

Among the lines measured for magnetic separation in this publication, the ratio of enhanced to non-enhanced lines is about 1:14 for iron and 1:5 for titanium. There appears to be no undue proportion of any one type of separation among the enhanced lines considered as a whole. None of the lines unaffected by the magnetic field are enhanced, and only one quintuplet occurs in this class. The more common types are all well represented among them. However, one noteworthy peculiarity is shown by the enhanced lines of titanium in the region from λ 3600 to λ 4600, rich in this kind of lines. Twenty-two lines occur here which show a high degree of enhancement, and are, as a rule, much stronger in the spark than any of the lines characteristic of the arc. Seventeen of these lines are clear triplets, while in the whole titanium spectrum measured, only a little over half of the lines are clear triplets. The remaining five, which are with one exception the weakest in the list, are of more complex character. As regards magnitude of separation, none of these 17 lines are normal triplets, and they appear to be less closely related to the interval *a* than a like number of triplets taken at random. These data are all presented in tabular form in the complete publication.

COMPARISON OF THE RESULTS FOR THE ZEEMAN EFFECT AND FOR PRESSURE DISPLACEMENT

This subject was treated for the spectra of iron, titanium, and chromium in a former paper.[1] The material for the first two elements is now presented with revised measurements and much more extensive data for titanium, for which I am indebted to Mr. Gale,

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 46; *Astrophysical Journal*, **31**, 433, 1910.

photographs having been made by him in this laboratory for the titanium arc under pressure. This extension gives 173 iron and 122 titanium lines available for a comparison of magnetic separation and pressure displacement.

The method of treatment of the material in tables is much the same as before, and similar conclusions are reached. The ratios of separation to displacement for different lines show no approach to constancy. In many cases the magnitudes of the two effects are very different, the extreme cases being lines unaffected by the magnetic field which show large pressure displacement. Taking the means of separation and displacement for large numbers of lines in the same region of the spectrum, however, it is found that these means are in general of the same order of magnitude when they are classified broadly as small, medium, and large. This shows that the lines as a rule show some degree of similarity in their response to the two displacing influences. Both effects are observed to increase with the wave-length, although it is not yet established that the rate of increase for the pressure-effect is the same as that for the Zeeman phenomenon, i.e., as the square of the wave-length.

MOUNT WILSON SOLAR OBSERVATORY
April 10, 1911

BODE'S LAW AND SPIRAL STRUCTURE IN NEBULAE

By WILLIAM SUTHERLAND

By the law of Titius, known as Bode's, with the mean radius of the earth's orbit as unit, the mean orbital radius of the planet which is nth from *Mercury* is

$$r=0.4+0.15\times 2^n. \tag{1}$$

For *Ceres*, as representative of the asteroids, $n=4$. As far as *Uranus*, for which $n=7$, the maximum difference between the observed values of r and those calculated by (1) is 5 per cent. In the case of *Neptune*, with $n=8$, the observed value of r is 30, while that calculated is 38.8, the difference rising to 29 per cent. We can write (1) in the form

$$r=r_1+r_2, \tag{2}$$

$$r_1=a=0.4, \tag{3}$$

$$r_2=be^{\theta/a}=0.15\times 2^n, \tag{4}$$

thus expressing that r can be obtained by adding to the radius vector of a circle of radius a the radius vector at angle $na\log_e 2$ of the logarithmic (equiangular) spiral (4). If we note that the circle is a limiting form of a spiral when a is infinite and θ is finite, we get the interesting mathematical result that the planetary distances are derived from two equiangular spirals, in one of which the constant angle between radius vector and tangent is $\pi/2$, and in the other the tangent of that angle is a. The law of planetary distances introduces us to a type of spiral simply related to the simple logarithmic spiral. So we are led to the physical hypothesis that the solar system in its evolution passed through the stage of a spiral nebula. Let us take the simplest case when $r_1=0$ and we consider the nebula whose spiral is (4).

The mean radius between $\theta=m\beta$ and $(m+1)\beta$ is

$$\bar{r}=\frac{1}{\beta}\int_{m\beta}^{(m+1)\beta} be^{\theta/a}d\theta=be^{m\beta/a}(e^{\beta/a}-1)a/\beta. \tag{5}$$

Thus in the planetary distances we can regard r_2 as derived from the breaking up of a logarithmic spiral nebula into equal

angular parts, that is, into parts which originally subtended the constant angle β at the origin. Equating coefficients in the second term of the right-hand side of (1) and (5) we get, when $m=n$, $e^{m\beta/a}=2^n$, $b(e^{\beta/a}-1)a/\beta=0.15$ $\therefore$ $\beta/a=\log_e 2=0.693$, and $b=0.15\times 0.693=0.104$. The breaking up of the original nebula into equal angular portions is suggestive of the existence of stationary vibrations in it. Let us consider the origin of these. Take to begin with a swarm of meteorites which has reached this sort of steady state, that it is no longer expanding. Gravitation is just sufficient to make the swarm cohere at its maximum size. It has lost all those meteorites which can escape from it. Gravitational collapse has just begun. If the velocities of the meteorites relative to the center of mass of the swarm are uniformly distributed in direction, the swarm will move on with uniform velocity of translation, without any prevailing rotation, and collapsing uniformly. These uniform conditions may not prevail throughout the swarm. Local condensations may be taking place, but on the whole there is no prevailing structure except that due to the inward motion of the parts collapsing. This is the simplest type of nebula.

But when the gravitationally cohering swarm has a prevailing rotation as well as translation, then that rotation prevents collapse in directions at right angles to its axis. In this case collapse goes on parallel to the axis and the swarm takes the form of a disk or lamina. We may start then with a spherical swarm as type of a uniform nebula and pass to the swarm typified by an oblate spheroid as representative of one in which a prevailing rotation destroys the symmetry with respect to the center and replaces it with symmetry in regard to an axis of rotation. When the oblateness of the spheroid is great, we reach the type of the disk nebula, which includes the great majority of the spiral nebulae. Now to the generalized swarm we can apply the principles of the kinetic theory of perfect gases. But when gravitational cohesion is taken into account, we pass to a case analogous to the kinetic theory of imperfect gases, in which cohesional force is producing measurable effects recognized in such an equation as that of Van der Waals. In this way we reach the conclusion that in a swarm of meteorites gravitational cohesion can produce effects like those

of liquidity and solidity in the general kinetic theory of matter. In a swarm of meteorites whose elements range from separate molecule to particle of dust and from particle of dust to star we have a large-scale rigidity and a large-scale bulk modulus (reciprocal of compressibility) connected with the gravitational energy and the kinetic energy of the elements of the swarm. In "The Electric Origin of Rigidity and Consequences" (*Phil. Mag.* [6], 7, 417, 1904) it is shown that at the absolute zero of temperature the rigidity of a solid is equal to the electrostatic energy per unit volume of its molecular electrization. At higher temperatures molecular kinetic energy plays a part. It seems to me then that in cosmical physics we must freely supplement the theory of three bodies with a theory of cosmic elasticity, with the aid of which to study the changes observed in nebulae regarded as solid and liquid bodies of which meteorites are the molecules. Stationary vibrations in a disk nebula are a dynamical necessity of this theory. The simplest type is that in which the disk vibrates in n equal segments with the radii as nodes. As to the causes which give n a definite numerical value I refrain from speculating at present. The separation into n vibrating segments determines the ultimate condensation into n planets in a manner which observation may elucidate.

Let us now trace the probable course of the development of spiral structure in a disk nebula. The most probable motion of a meteorite of mass m at distance r from the center is that in a circle with velocity v such that mv^2/r is equal to the gravitational attraction of the whole swarm toward the center. Near a circle of radius r we shall have two circular streams traveling in opposite directions with this velocity v. One stream is more massive than the other, whence the prevailing rotation of the whole disk, though we see that a disk could exist without a prevailing rotation, if it had equal opposite circular streams that failed to destroy one another by collisions. Where one stream is more massive than the oppositely directed one, we may regard it as containing more meteorites of the average size. Suppose two average meteorites moving in opposite directions collide so as to form a mass of velocity o. This at once proceeds to fall toward the center, describing

a radius. We may designate this radial path as a spiral of zero spirality, and the circular path as one of infinite spirality. We see that the radially falling body will get circular motion imparted to it by collision, and thus we arrive at an average path between circular and radial, namely a spiral of finite spirality. The meteorites of the collapsing swarm move in toward the center on spiral paths. If these are uniformly distributed round the center, they may give little or no evidence of spirality, they appear as simple disk nebulae of unknown spirality which may have any value between zero and infinity, a ring nebula perhaps making its infinite spirality known.

We have now to consider the remarkable fact that so many of the typical spiral nebulae consist of two spirals. Theoretically we can see how there is a tendency for tracks of greater meteoritic density to form. In the first place, gravitation helps any denser region to become more dense. In the second, collisions are more frequent in the denser region, making its spirality smaller, so that it cuts the less dense tracks of larger spirality, robbing them of meteorites by reason of the more frequent collisions and the stronger gravitation. In these ways the dense region generally tends to become more dense. This is only a particular case of the universal tendency under gravitational attraction. We see that there is a process by which the spiral tracks coalesce into fewer ones of higher meteoritic concentration. But we should expect the number of spirals to be determined by the initial departures from uniformity of the distribution of the meteorites in the swarm. It is probable that, as the central body grows by the inflow of meteorites along the spirals, it creates dynamical conditions favorable to the reduction of the spirals to two main ones symmetrically placed with regard to itself. In some photographs of spiral nebulae there are signs of a spiral breaking into branches, which are probably remnants of spirals coalescing into one. Two main spirals represent the simplest symmetrical arrangement by which momentum in the spiral paths can be transmitted to the rotating central body. No doubt it is a very complex problem to find the definite dynamical reason for the existence of the spirals so frequently as symmetrical pairs.

So far we have reached the conception of a disk nebula rotating as a whole round its center, while there is a streaming motion of its constituent parts inward toward the center along spiral paths which generally reduce to two. With a scale large enough in space and time we may regard it as a solid body rotating, undergoing deformation, and flowing like a liquid along the spirals, just as ordinary solids flow under suitable stresses, while at the same time the whole elastic structure is vibrating. In studying the photographic records of nebulae it will be necessary to bear in mind these general modes of motion. An important dynamical point is this: Does the disk nebula, while it concentrates into two spirals, still vibrate as a disk divided into equal segments by radiating nodal lines? We are led to suppose that it does, to account for the derivation of each planet being from an arc of the spiral subtending a constant angle at the center. So the evolution of a planet must now be discussed. The vibrational movement of each internodal segment of a spiral will be damped down by internal friction, that is to say, by collisions among meteorites. So frictional or viscous damping on the cosmic scale means coalescence of meteorites. Collisions aided by gravitation favor aggregation in each vibrating segment. But as the meteorites gather into larger and larger masses, into incipient planets, they move in a region where the meteorites are less dense. The planetary nucleus tends to move independently of its segment of the spiral, to take to a path of its own of larger spirality, cutting periodically the region near the middle of its segment of the old spiral and growing at its expense, while describing an orbit that becomes more and more nearly a circle round the center. Meanwhile other meteorites have continued to stream along the old spiral tracks into the center. Sun and planet grow simultaneously, the sun being indeed only the large central planet. If the solar system consisted at one stage of two main spirals, then the corresponding internodal segments of each must have built up together a single planet. According to this scheme of evolution it is probable that the moon and the other satellites originated as small subsidiary or branch spirals such as we see in the photographs of spiral nebulae.

A very remarkable fact about the planets is that *Mercury*, with

the smallest mass, namely 0.074 of the earth's, comes next to the sun, which has the vast central mass, while *Mars*, with the next smallest, namely 0.134, comes almost next to *Jupiter*, the largest, of mass 343, the asteroids of very small mass coming between. The special enrichment of the sun and *Jupiter* and *Saturn* meant the impoverishment of the planets near them, *Mercury*, *Mars*, the asteroids, *Uranus* and *Neptune*. At first sight it appears as if these large contrasts in the masses of neighbor planets make it very improbable that the planets originate in internodal segments. One might naturally expect nodal segmentation to give rise to planets of nearly equal masses. But further consideration presents the matter in a new light. Let E be a modulus of cosmic elasticity of the matter of a nebula, ρ its density cosmically measured, then the velocity of propagation of a disturbance through the nebula is $(E/\rho)^{\frac{1}{2}}$. If E is proportional to ρ, this velocity is independent of e density in any part of the nebula, so that the disk nebula is divided up by its nodes into segments without reference to the distribution of density in the nebula. A similar case occurs with the velocity of sound in any given gas, which is independent of its pressure. If the earth's atmosphere were at a uniform temperature, its nodes of vibration would be distributed irrespective of the variation of the density of the air with altitude. When E is proportional to ρ, we may infer that the energy giving rise to the modulus E at any point in the nebula is due to the mutual gravitational energy of unit volume of the nebula at that point and the rest of the nebula, which at the stage we are considering will be nearly the mass of the central sun. In that case the gravitational energy of unit volume will be proportional to its density, and also inversely proportional to the distance of the point from the center of the sun. Parts of the nebula at the same distance from the center have E proportional to ρ. The velocity of propagation of a disturbance round a circle of radius r is constant, however the density varies along the circle. It follows then that radial nodal lines will be distributed at equal angular intervals without reference to the distribution of density in the nebula after most of its matter has gathered at the center. The contrast in the masses of neighbor planets causes no difficulty in the theory that the planets

originate through the cosmical music of the nebula, "the harmony of the spheres." In the speculations of the Greeks this harmony was a harmonical spacing. To account for the contrasts a marked want of uniformity in the distribution of the meteorites in the swarm must be postulated, gravitation accentuating this by increasing the store of the denser regions at the expense of the less dense.

At the present time, thanks to recent advances in astronomical photography, we can investigate the spirality of typical spiral nebulae in a fairly definite manner, though concerning their linear dimensions we have little accurate knowledge. We recognize that the size of a nebula has a vast range if we proceed from the rings of *Saturn* to the great nebula in *Andromeda* having an angular length of 2°.5. But just as the broad elastic laws of a grain of sand are the same as those of a mountain of quartz, and those of a speck of jelly the same within the right limits as those of an ingot of steel, the laws of cosmic elasticity must produce similar results in *Saturn's* rings to those in the largest nebulae. So no difficulty arises in comparing the primitive solar nebula with the vaster ones of the heavens except the practical one of measuring the linear dimensions of these remote bodies. But in the measurement of the spirality of nebulae we can obtain at once satisfactory results from the photographs. The chief uncertainty in these arises from the necessity of assuming that the lines of luminosity in a nebula show the lines of greater density. With this assumption we can obtain the spirality of typical nebulae from their photographs. As the most favorable example we may take Ritchey's photograph of the spiral nebula *Messier 51 Canum Venaticorum*, reproduced in *Nature*, **85**, 142, from this Journal, Plate IV, **32**, 1910. The following are the values of the radius vector in mm for values of θ increasing by $\pi/2$.

θ	0	$\pi/2$	$2\pi/2$	$3\pi/2$	2π
r in one spiral	7	12	18	25	
r in the other	7	11	19	28	60
r mean	7	11.5	18.5	26.5	60
r calculated	6.2	10.7	18.6	32.4	56

The calculated values of r are obtained from the equation

$$\log_{10} r = 0.79 + 0.24(2\theta/\pi) \text{ or } \log_e r = 1.819 + \theta/2.84, \qquad (6)$$

constructed as the best logarithmic spiral to represent the relation

between the mean r and θ. The data are hardly definite enough to warrant the attempt to treat r as r_1+r_2. The spirals of this nebula can be fairly well represented by logarithmic spirals of spirality $a=2.84$ radians. Two other photographs suitable for similar measurements are those of Nebula *M 100 Comae* by Roberts and of Nebula *M 101 Ursae Majoris* by Ritchey, reproduced in Lowell's *Evolution of Worlds*, the results being subjoined.

	Nebula *M 100 Comae*				Nebula *M 101 Ursae Majoris*			
θ	0	$\pi/2$	$2\pi/2$	$3\pi/2$	0	$\pi/2$	$2\pi/2$	$3\pi/2$
r in spiral 1	4	5	8	11	4	5	6	9
r in spiral 2	..	5	9	..	4	5.5	8	..
r mean	4	5	8.5	11	4	5.2	7	9
r calculated	4	5.6	8	11.2	4	5.2	6.9	9.1

The equations used in calculating r to represent the mean r were

$$\log_e r = 1.38+\theta/4.55 \text{ and } 1.38+\theta/5.69. \qquad (7)$$

Here again we find that the spirals can be regarded as approximately logarithmic. The spiralities of these three nebulae, measured by a in radians, are 2.84, 4.55, and 5.69, that for the ring nebulae being ∞, with 0 for the case of a uniformly collapsing spherical nebula. For the case of the primitive solar nebula we did not obtain a, the spirality, but only $\beta/a=0.693$, β being the unknown angle subtended at the center by the segment of the spiral from which each planet was derived. As the visible range of θ in the three nebulae considered above does not exceed 2π, and as there are 8 planets, we may take the order of magnitude of β to be given by $2\pi/8$. With this value for β in the solar nebula a becomes 1.13, and the spirality of the solar nebula ranges itself naturally with those for the three nebula taken as typical.

It is interesting to recall the fact that the logarithmic spiral is the orbit of a particle moving under an attraction varying inversely as the cube of the distance from the center. It appears then that, after the greater part of a nebula has gathered into a central star, we may regard the larger part of the gravitation as producing cohesion of the whole and along with the general rotation providing the general rigidity, while at the same time part of the attraction

of gravity is so modified by collisional viscosity as to operate like an unresisted attraction varying inversely as the cube of the distance from the center.

A rather important point in observational astronomy raised by the present theory is the desirability of searching systematically for uncondensed vestiges of the original solar nebula not only near the sun but out as far as *Neptune*. Does the earth at intervals of nearly a year pass through the remnant of the segment of the spiral or each of the two spirals from which it was condensed? Two crucial observations needed to verify the present theory are that of inward motion along the spirals of all spiral nebulae and the demonstration of a process of regular segmentation of the spirals in some of them.

MELBOURNE, AUSTRALIA
April 1911

POSTSCRIPT added July 27.—The trend of previous thought concerning a connection between the structure of the solar system and that of spiral nebulae may be indicated by two references: first, to William Thomson's summary of three papers of 1854 in Vol. II, art. lxvi, of his *Mathematical and Physical Papers*, where, in discussing the origin of the sun's heat from the falling of matter into it, he states that a metoer would approach the sun by a very gradual spiral; and, second, to F. R. Moulton's paper in this JOURNAL, **22**, 165–181, 1905, on Chamberlin and Moulton's theory of the development of the solar system from a spiral nebula.

SECOND POSTSCRIPT.—August 3: Having seen today an article in *Astronomische Nachrichten*, No. 4503 (**188**, 250, 1911), by E. v. d. Pahlen entitled "Ueber die Gestalten einiger Spiralnebeln," I can add the following information. This author finds that selected nebulae have their shapes definite enough to be taken as logarithmic spirals. He assumes that the selected spirals lie each in a plane, and considers the case where this plane is not at right angles to the line of sight. He takes the spirals of *M 51 Canum Venaticorum* to lie in a plane making an angle of 42° with the plane at right

angles to the line of sight, and finds spiralities which are 4.28 and 4.24 when expressed in radians. These are markedly larger than the mean value 2.84 which I have given above on the assumption that the plane is at right angles to the line of sight. He finds that *M 33 Trianguli* and *M 74 Piscium* are at right angles to the line of sight, and have spiralities 1.16, 1.25, and 4.08, 3.53. The values 1.16 and 1.25 for the two spirals of *M 33 Trianguli* are close to 1.13 estimated above for the solar nebula. He finds the spirals of other nebulae to be logarithmic, but does not give the coefficients from which their spiralities can be calculated.